Team of
AUSTRIA

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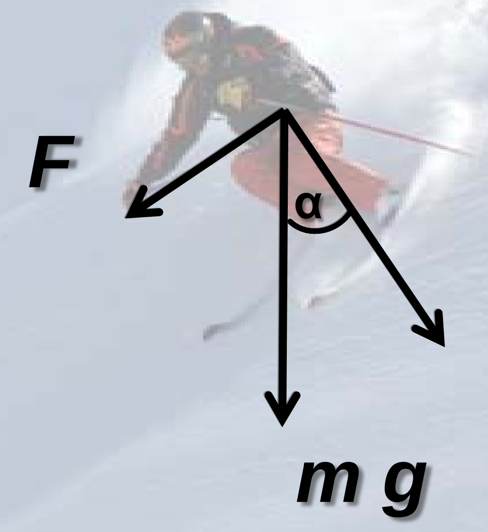
Bernhard Zatloukal

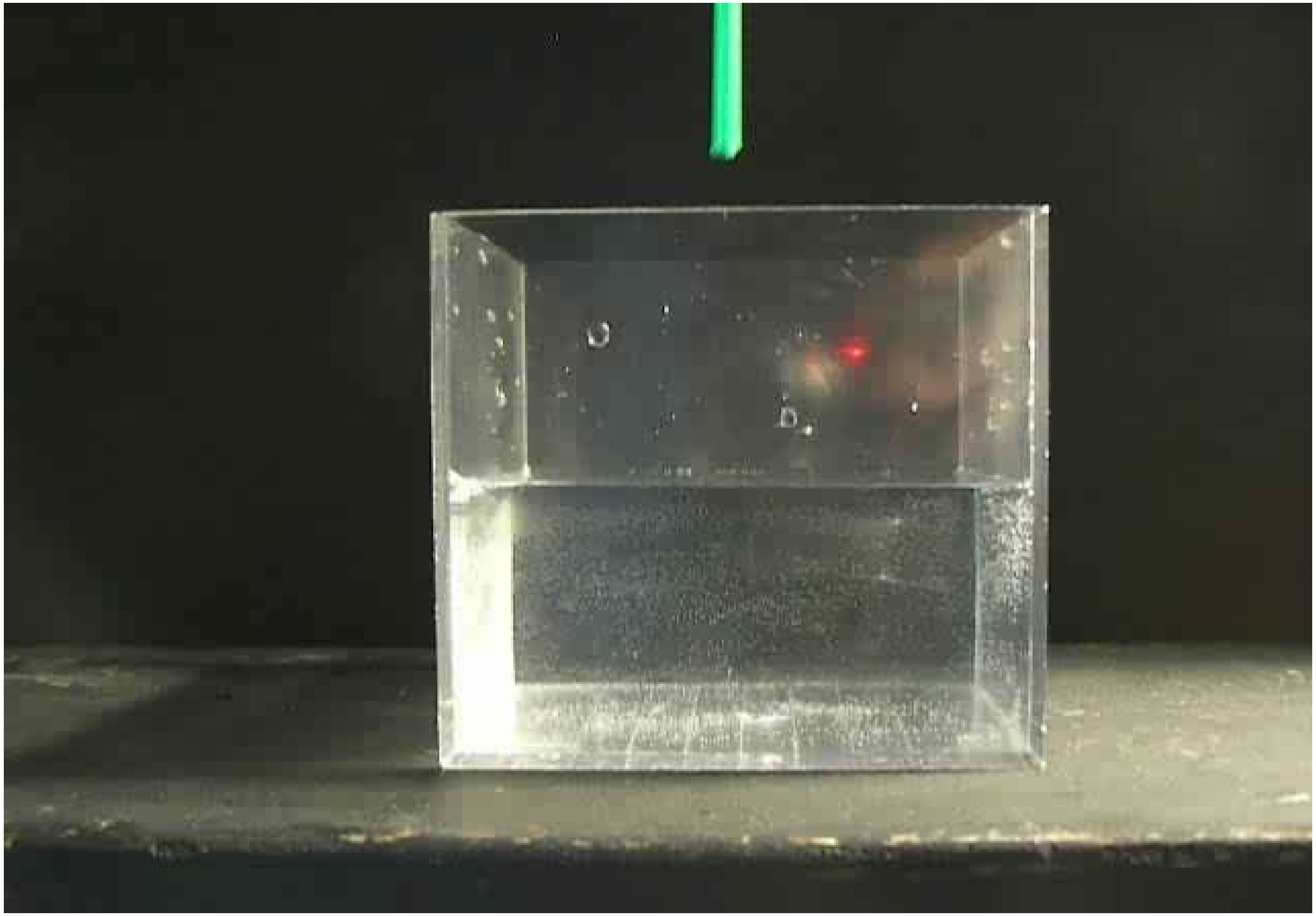
Presented by Angel Usunov

8. Air Pocket:
A vertical air jet from a straw produces a cavity on a water surface. What parameters determine the volume and depth of the cavity?



IYPT 2009 – Tianjin, China

Preliminary experiment



Overview

Theoretical approach

- 1st approximation
- 2nd approximation
- 3rd approximation

Experiments & comparison with our theory

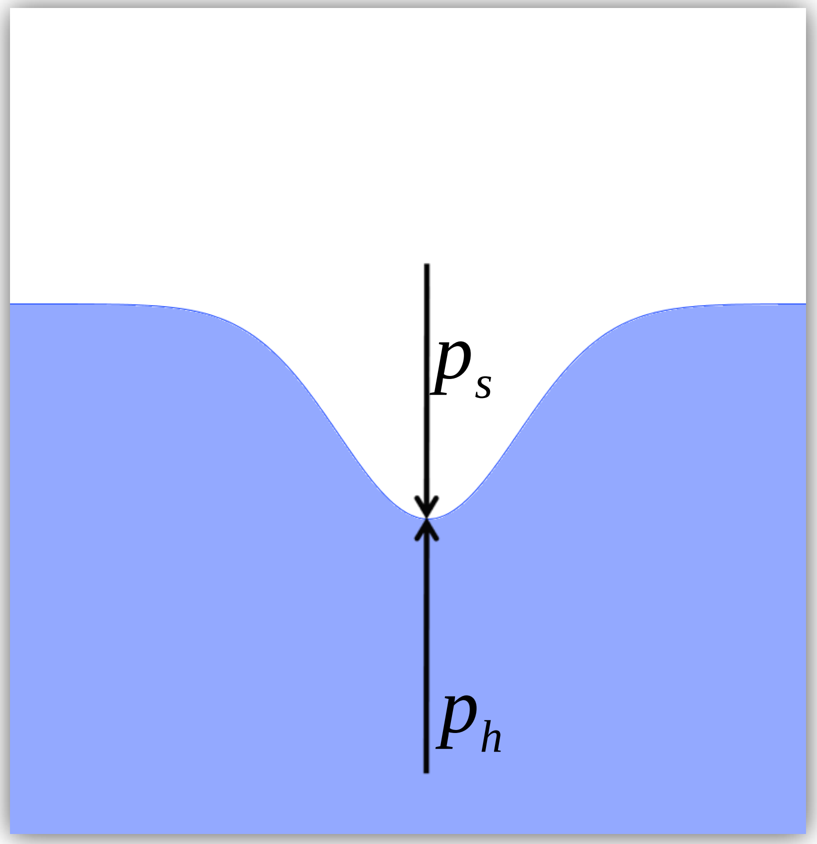
- Cavity depth & volume
- Cavity shape

Extending the model

- Flow analysis
- Surface tension

Conclusions

Theoretical approach - depth



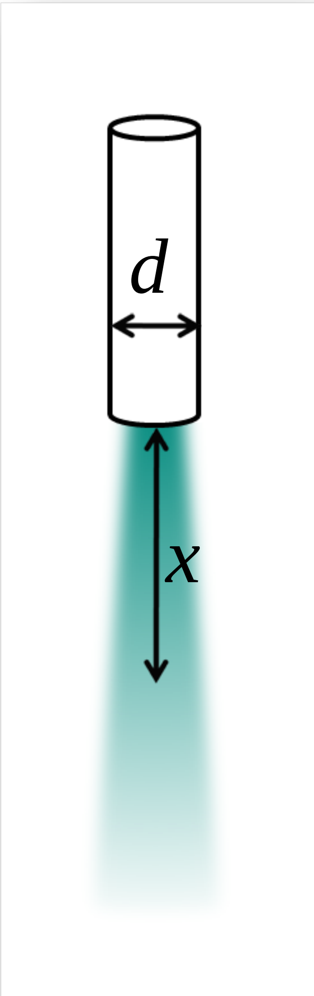
$$p_s = \frac{1}{2} \cdot v_j^2 \cdot \rho_g \qquad p_h = n_0 \cdot g \cdot \rho_W$$

$$p_s = p_h$$

$$\frac{1}{2} \cdot v_j^2 \cdot \rho_g = n_0 \cdot g \cdot \rho_W$$

$$\boxed{n_0 = \frac{v_j^2 \cdot \rho_g}{2 \cdot g \cdot \rho_W}}$$

Theoretical approach – depth

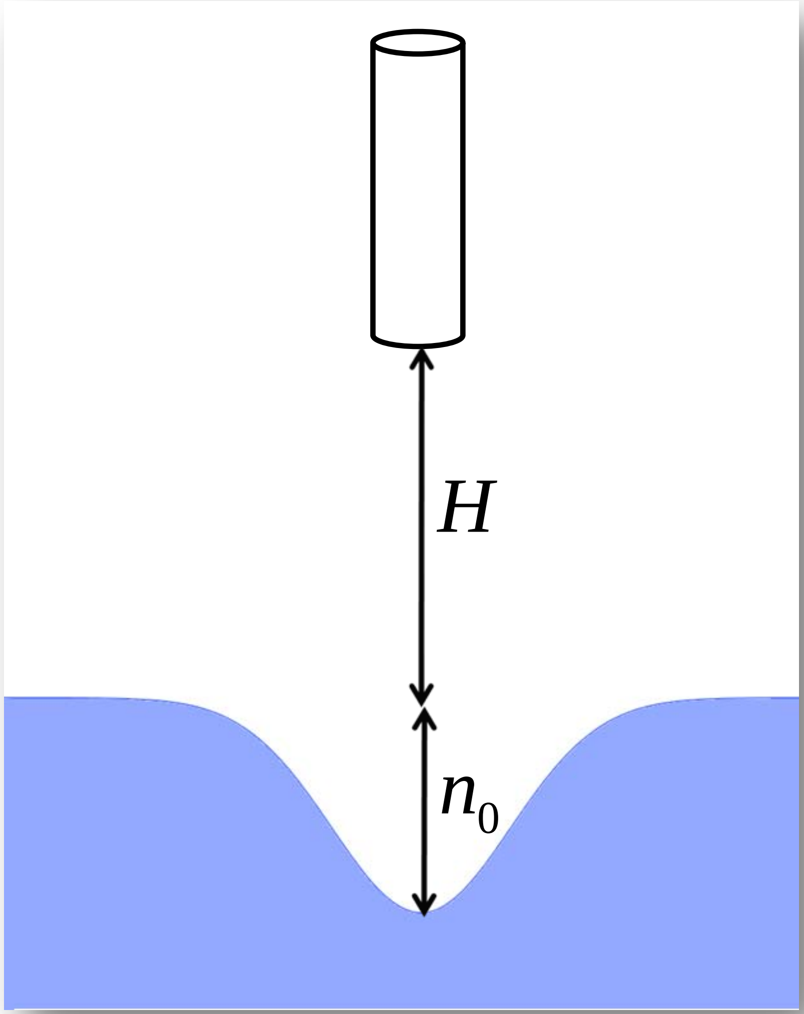


$$v_{cent} = v_j \cdot K \cdot \frac{d}{x}$$

$$v_{cent} = v_j \cdot K \cdot \frac{d}{H}$$

$$n_0 = \frac{v_j^2 \cdot \rho_g}{2 \cdot g \cdot \rho_W} \Rightarrow n_0 = \frac{\left(v_j \cdot K \cdot \frac{d}{H}\right)^2 \cdot \rho_g}{2 \cdot g \cdot \rho_W}$$

Theoretical approach – depth

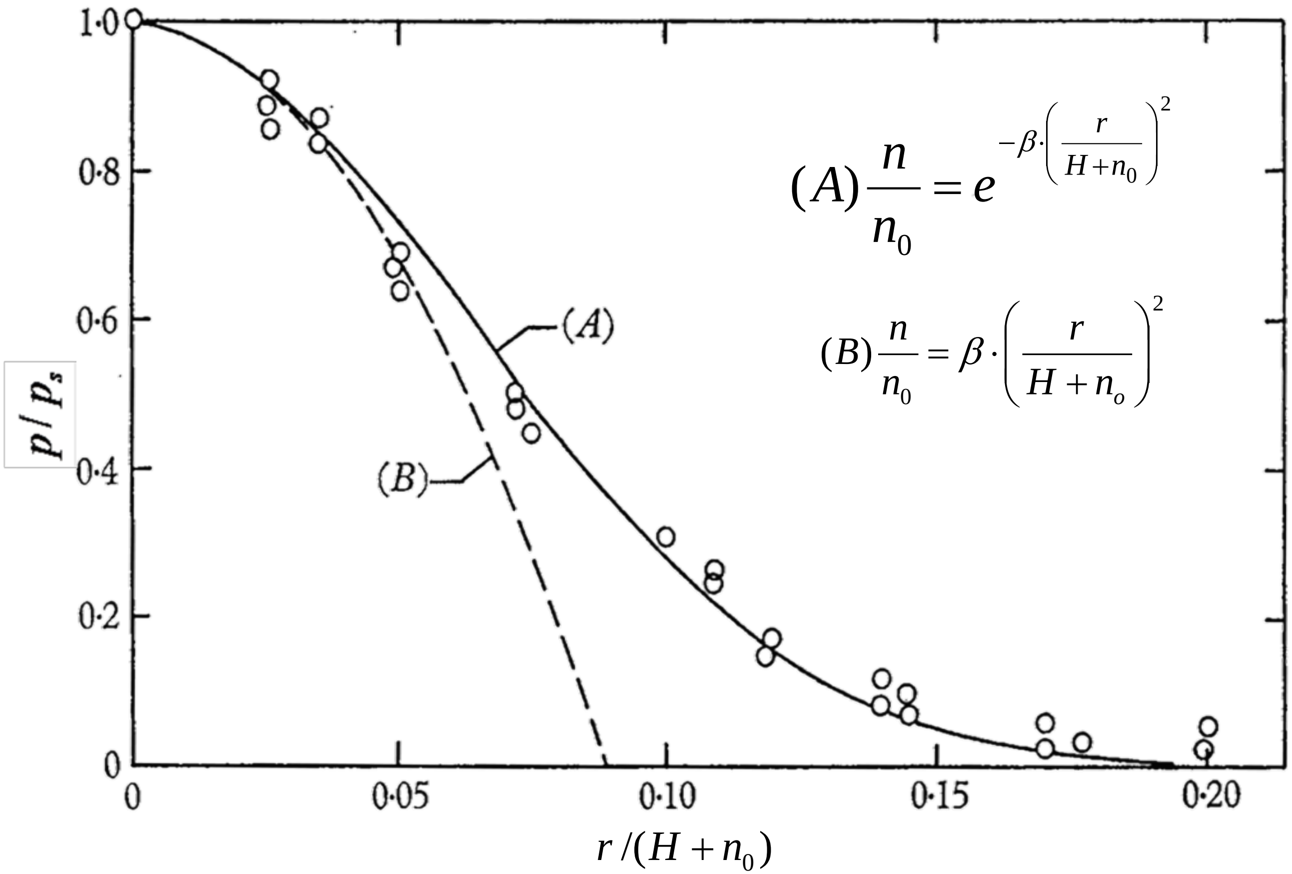


$$n_0 = \frac{\left(v_j \cdot K \cdot \frac{d}{H}\right)^2 \cdot \rho_g}{2 \cdot g \cdot \rho_W}$$

$$n_0 \cdot \left(n_0 + H\right)^2 = \frac{v_j^{\,2} \cdot K^2 \cdot d^2 \cdot \rho_g}{2 \cdot g \cdot \rho_W}$$

Theoretical approach – volume

- Pressure distribution determined by Poreh & Cermak (1959)



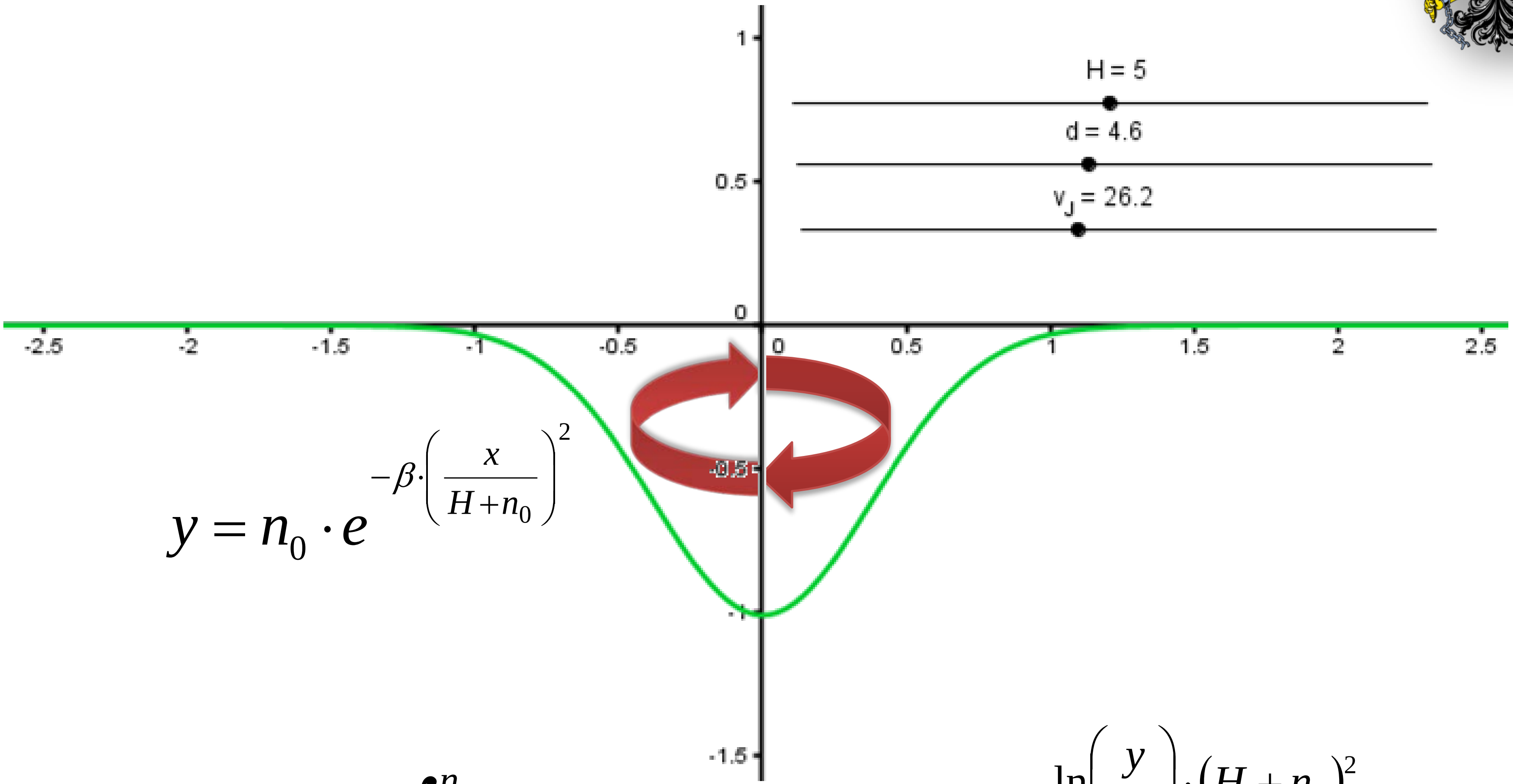
$$(A)\ \frac{n}{n_0} = e^{-\beta\cdot\left(\frac{r}{H+n_0}\right)^2}$$

$$(B)\ \frac{n}{n_0} = \beta\cdot\left(\frac{r}{H+n_o}\right)^2$$

$$\beta = 125$$

- Pressure at each point put equal to the hydrostatic pressure

Theoretical approach – volume



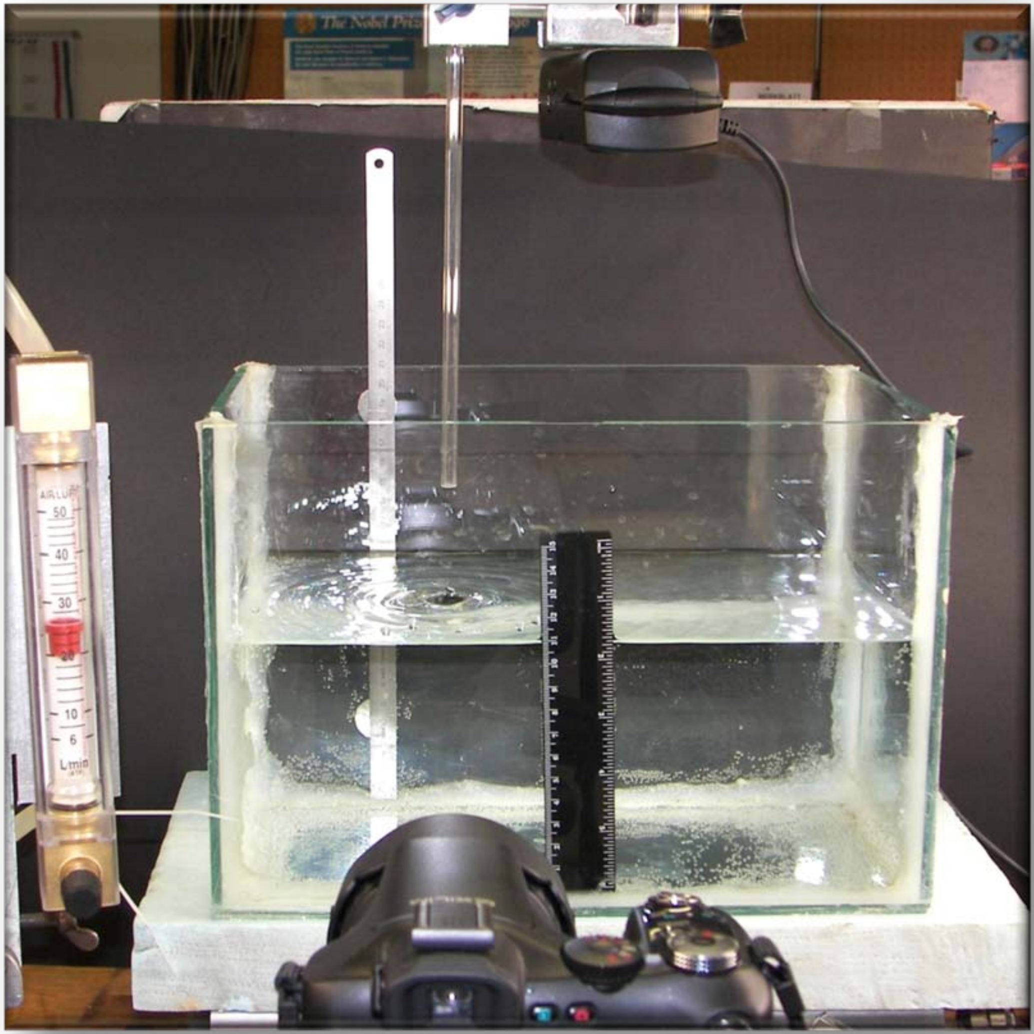
$$y = n_0 \cdot e^{-\beta \cdot \left(\frac{x}{H+n_0}\right)^2}$$

$$V_y = \pi \cdot \int_0^{n_0} x^2 dy$$

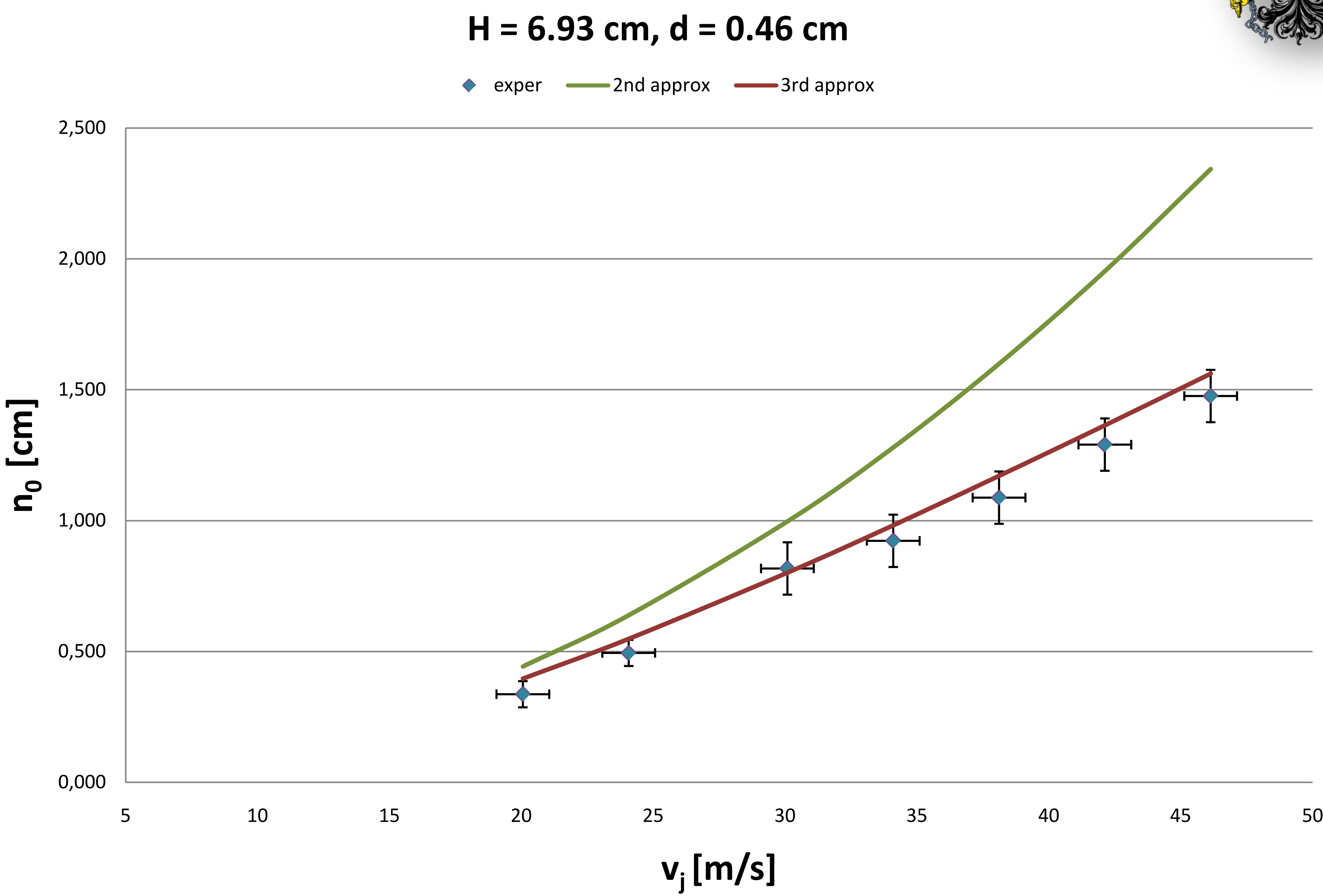
$$x^2 = \frac{\ln\left(\frac{y}{n_0}\right) \cdot (H+n_0)^2}{-\beta}$$

Experimental Setup

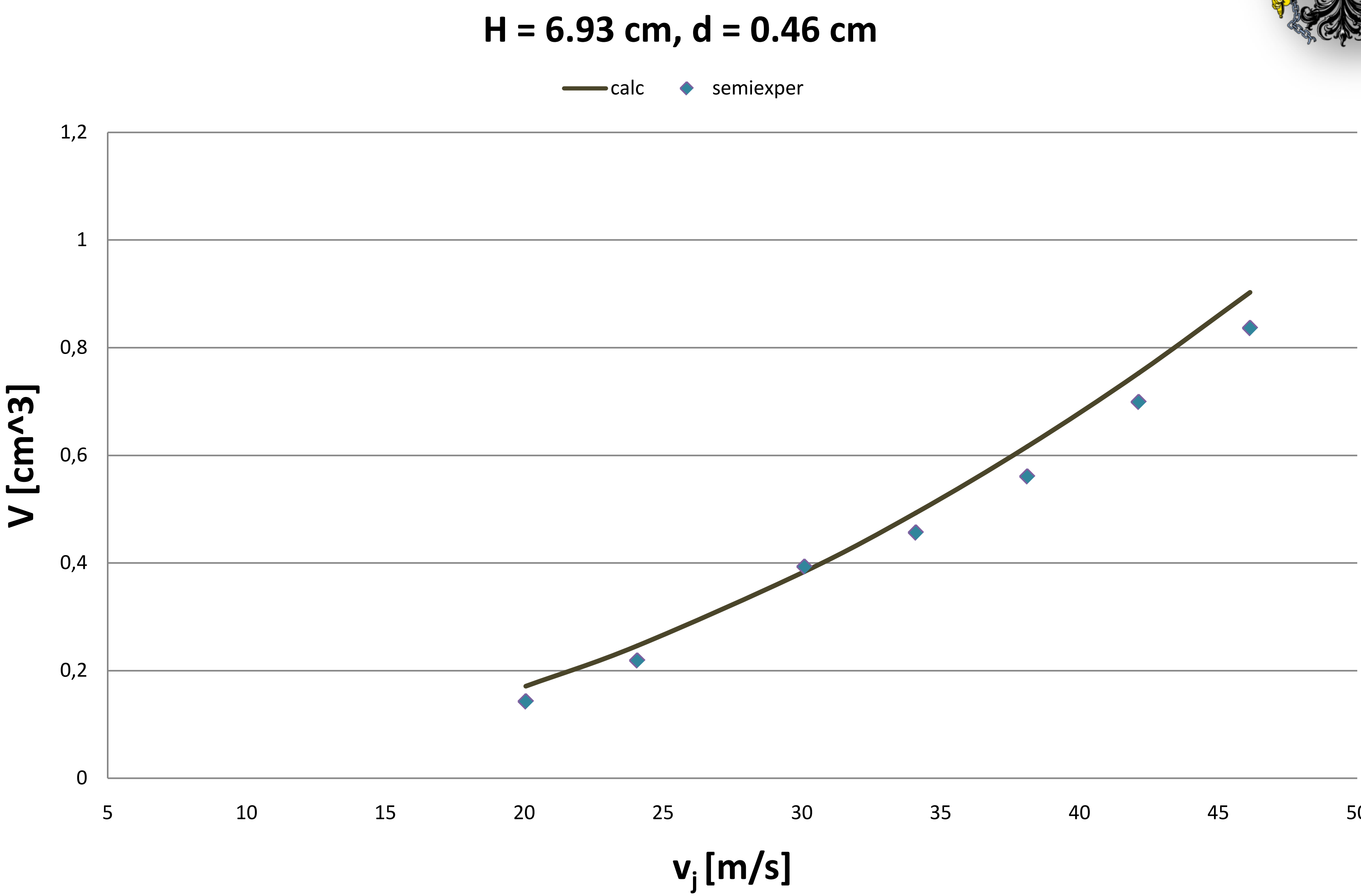
- Variable parameters:
  - gas flow rate
    - 6-50 L/min
  - tube diameter
    - 4.6mm; 3.5mm; 6.65mm
  - distance between the tube and the water surface
    - 1-9 cm



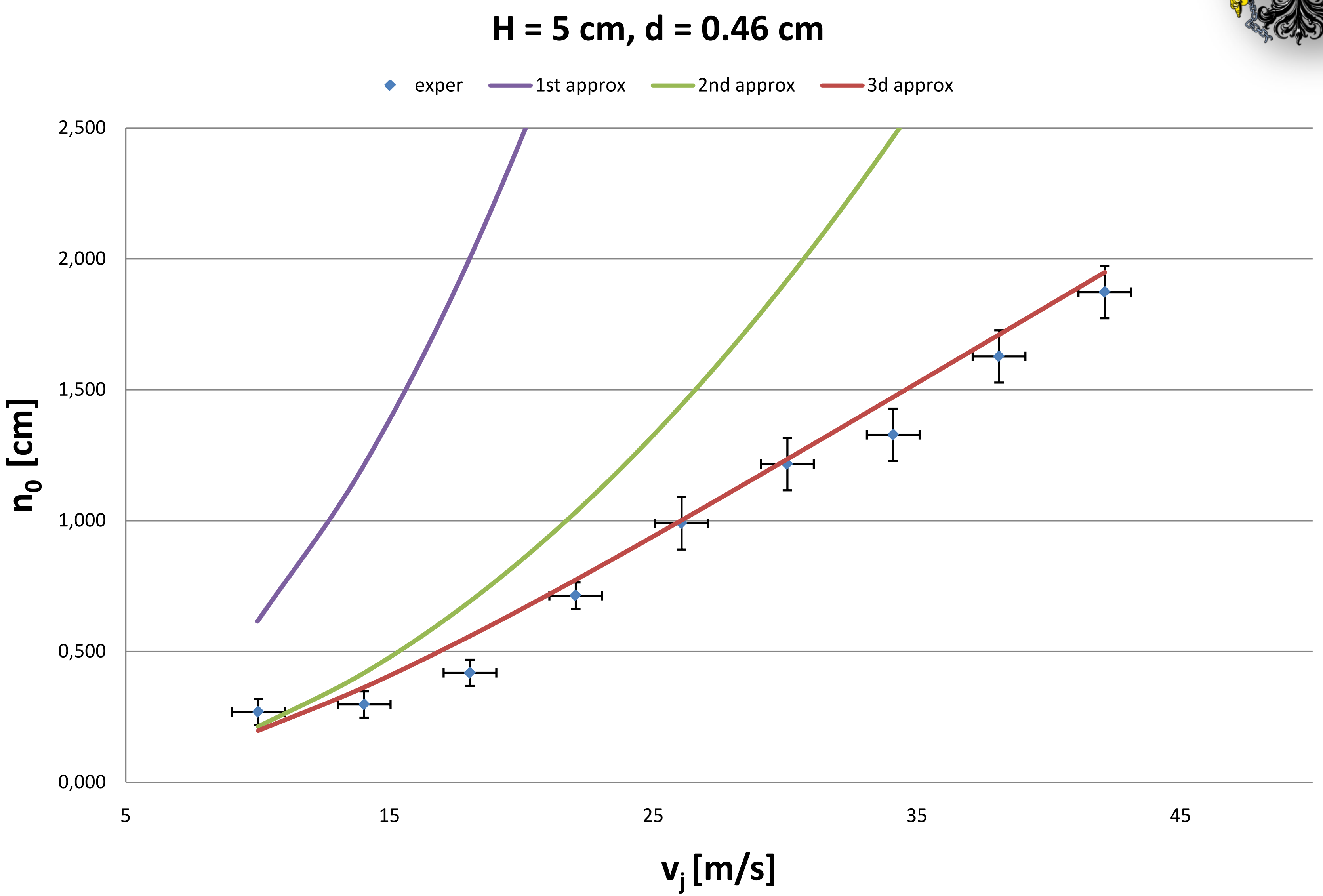
Experimental results – depth



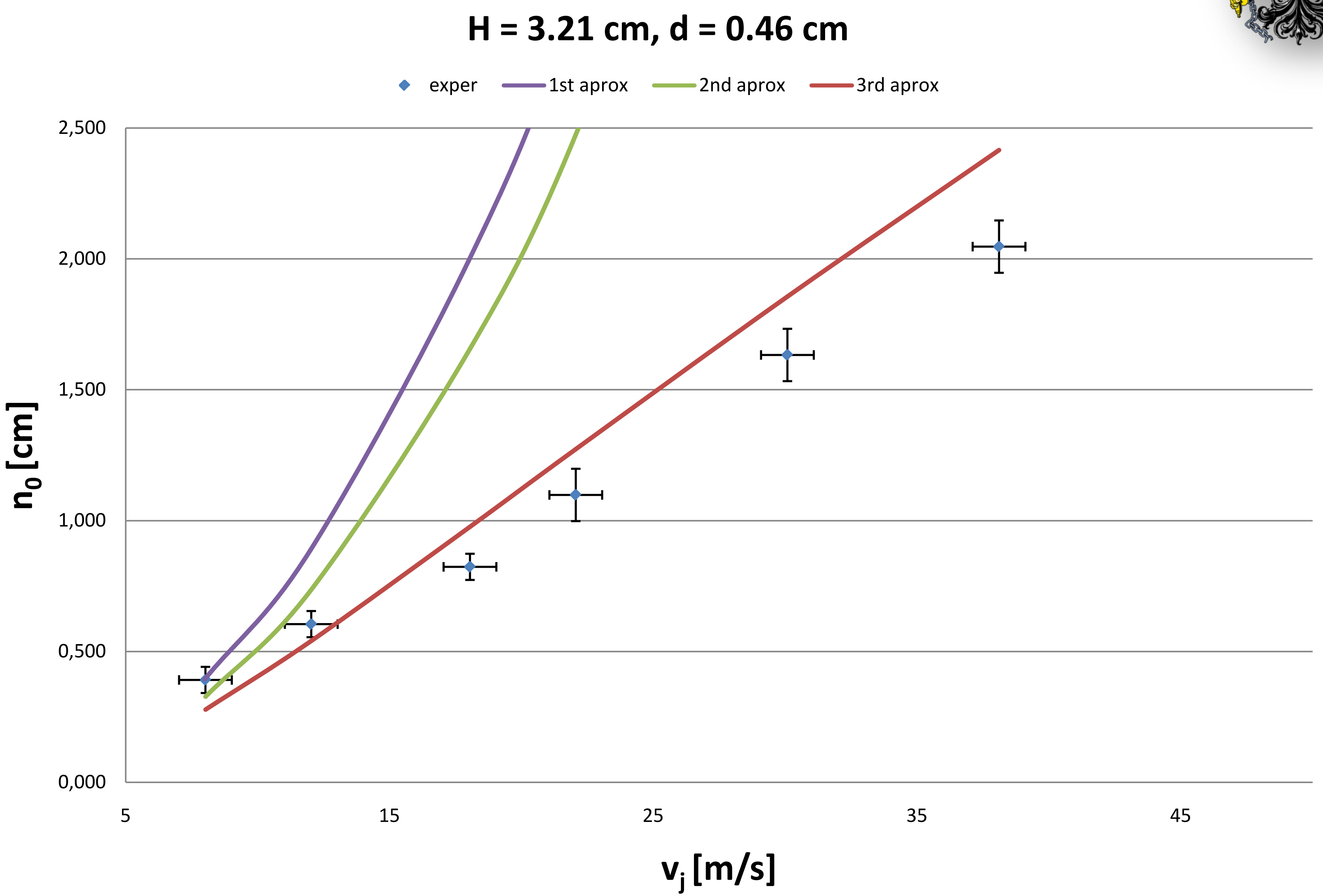
Experimental results – volume



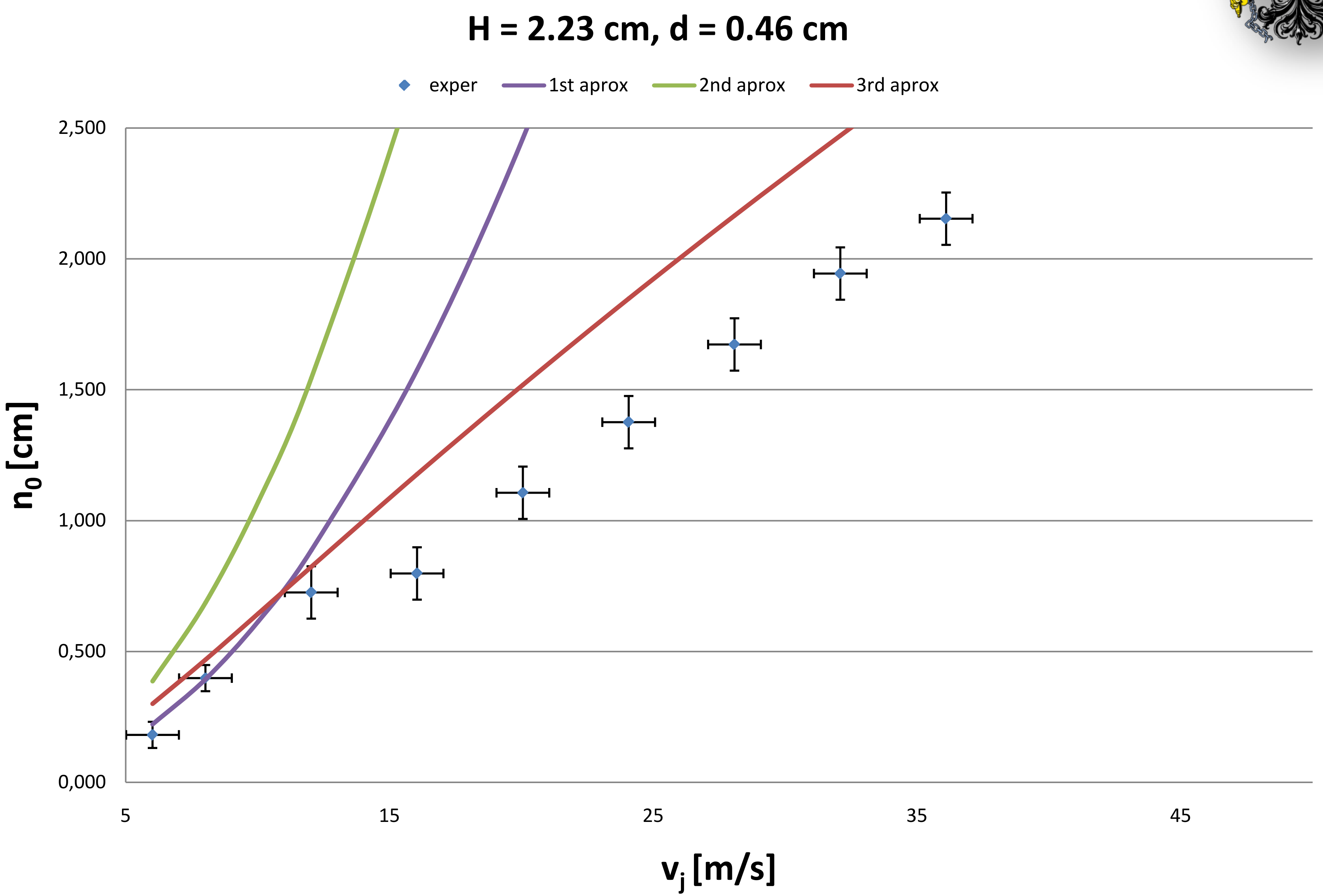
Experimental results – depth



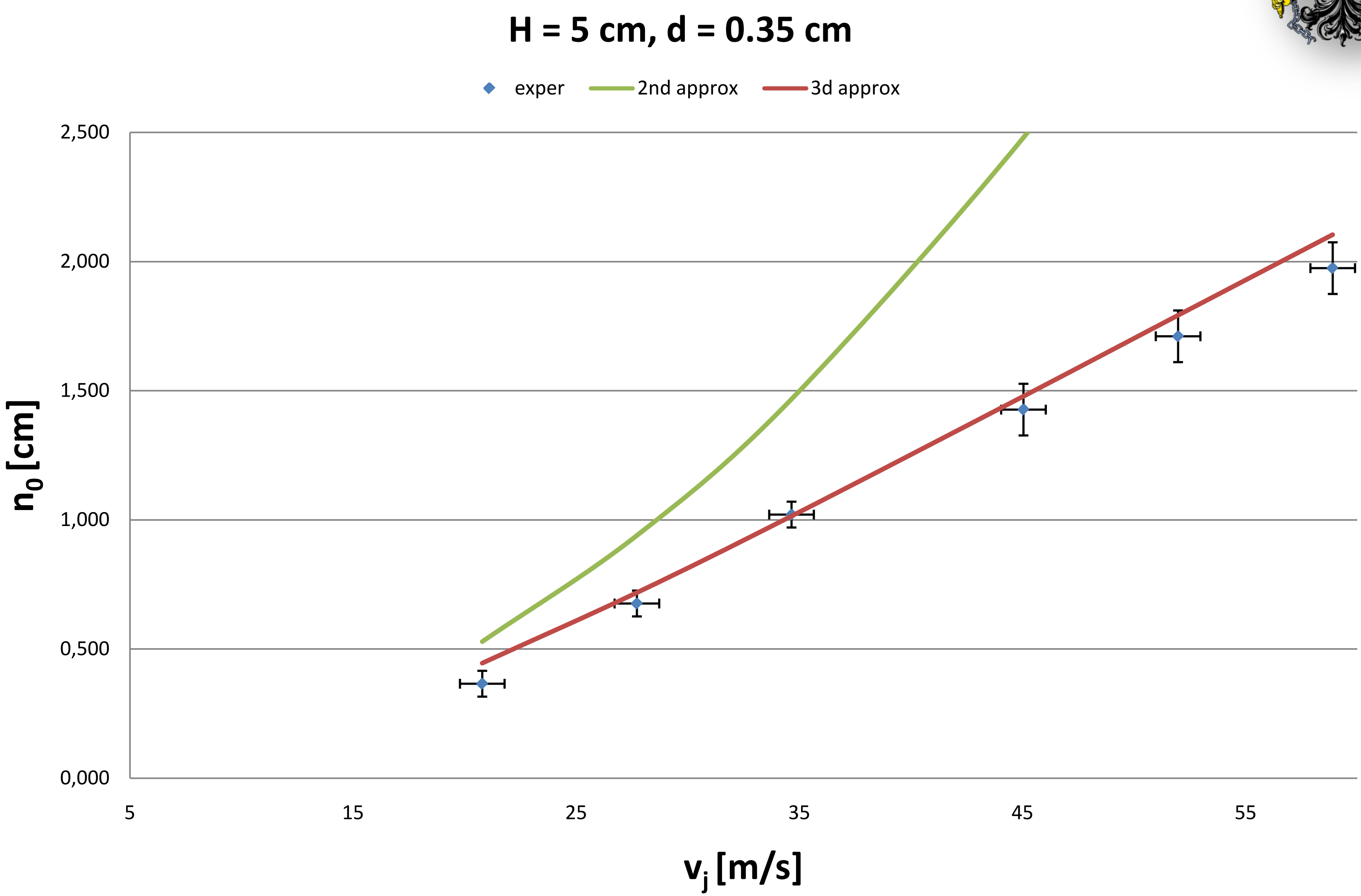
Experimental results – depth



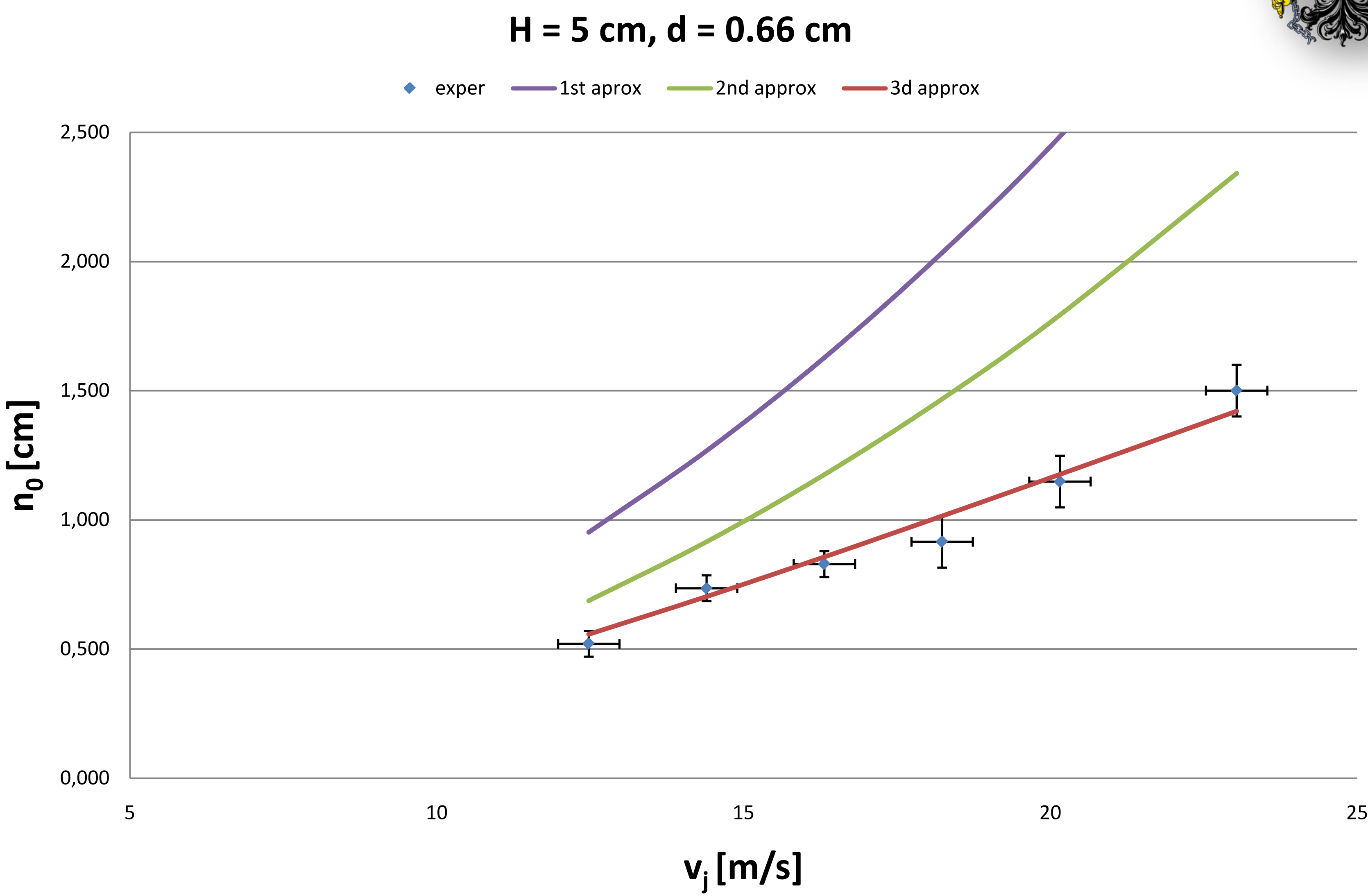
Experimental results – depth



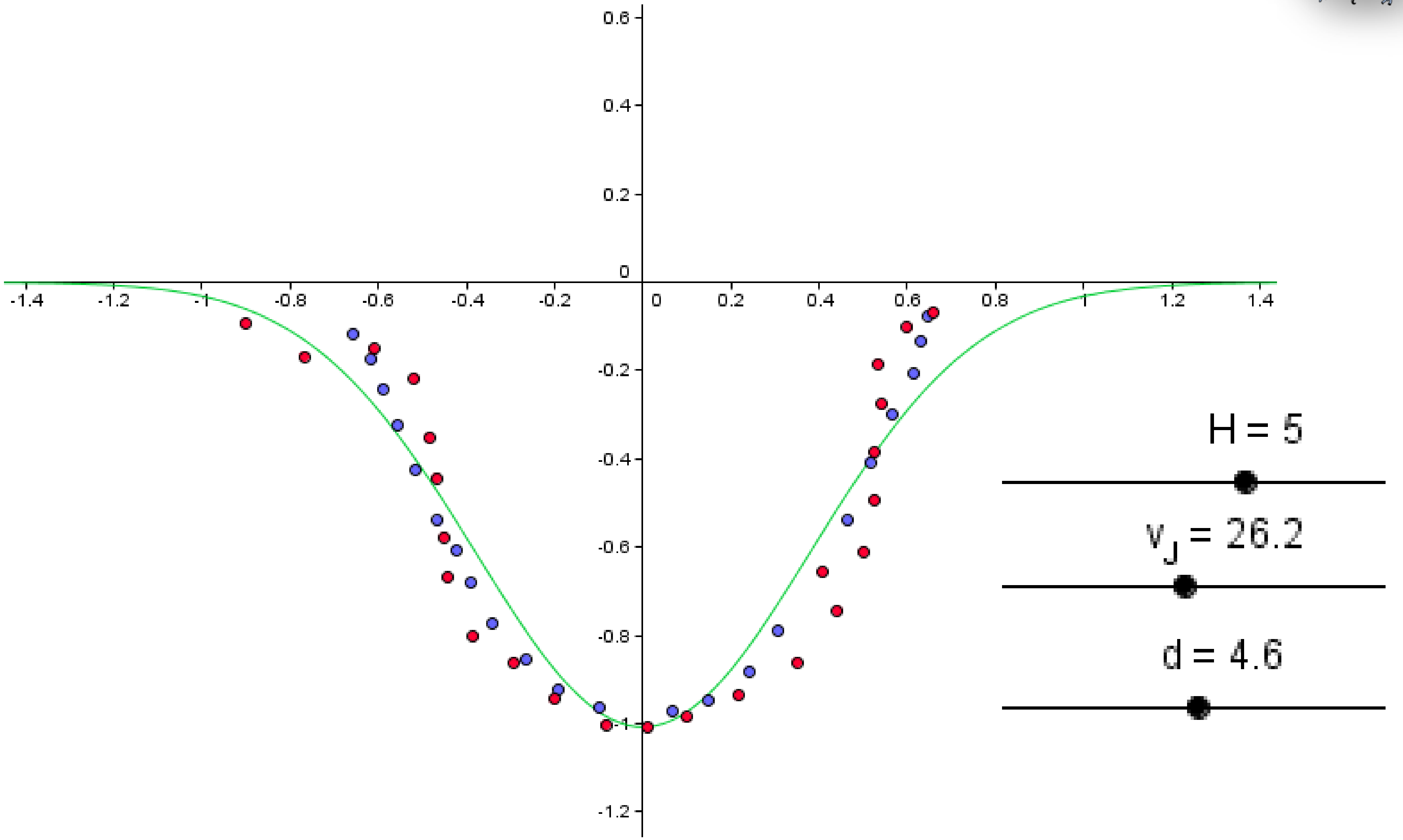
Experimental results – depth



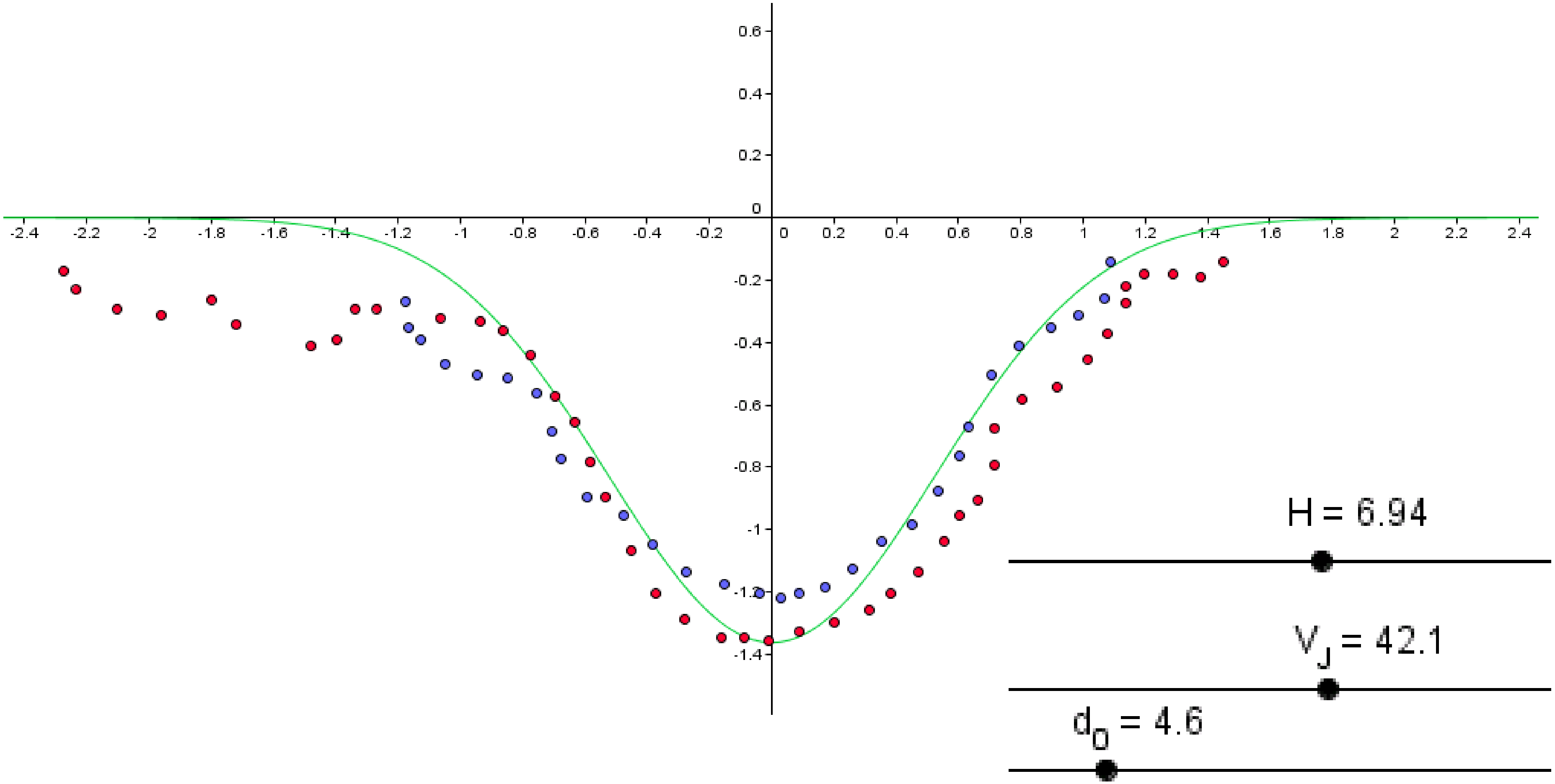
Experimental results – depth



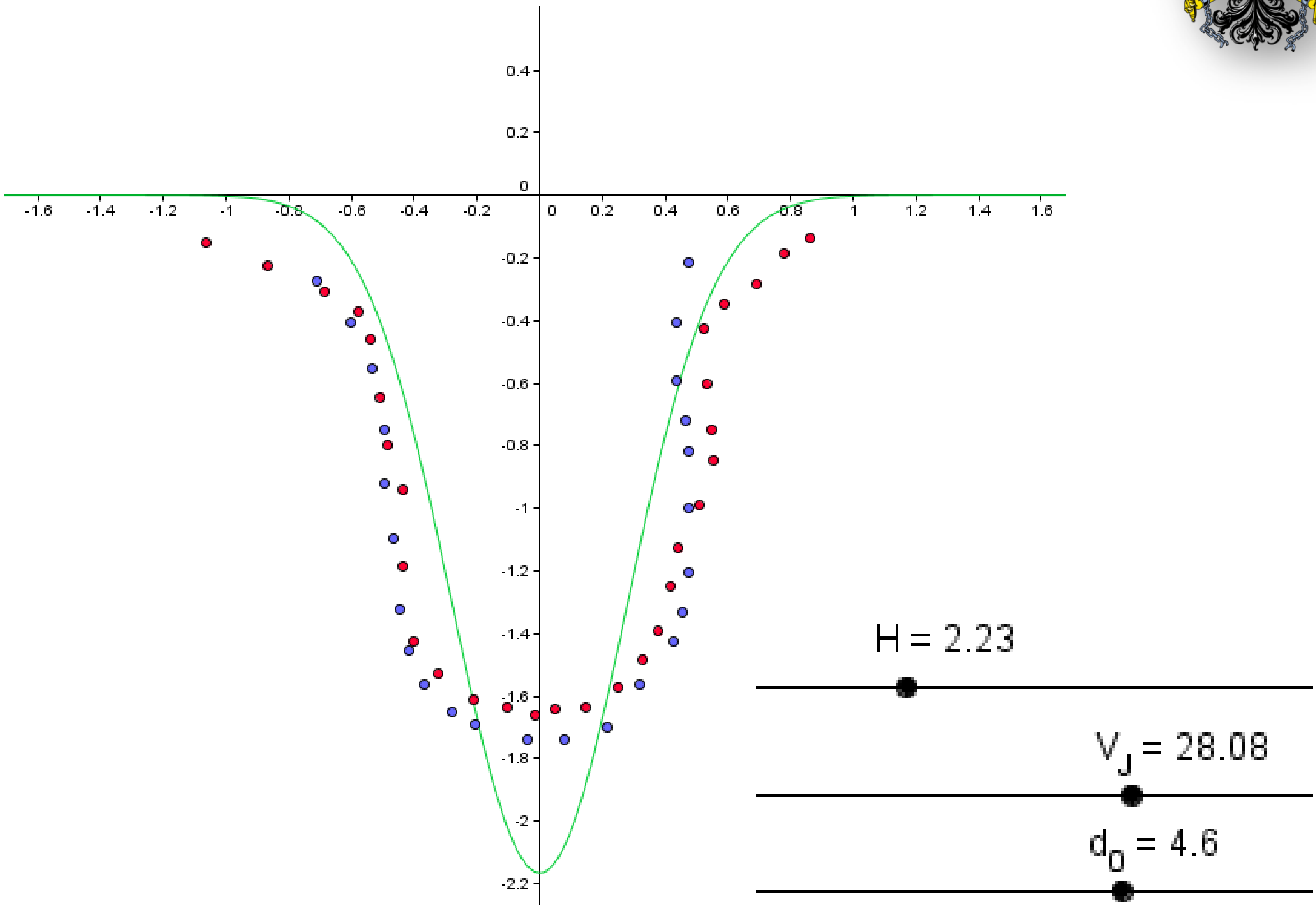
Experimental results – shape



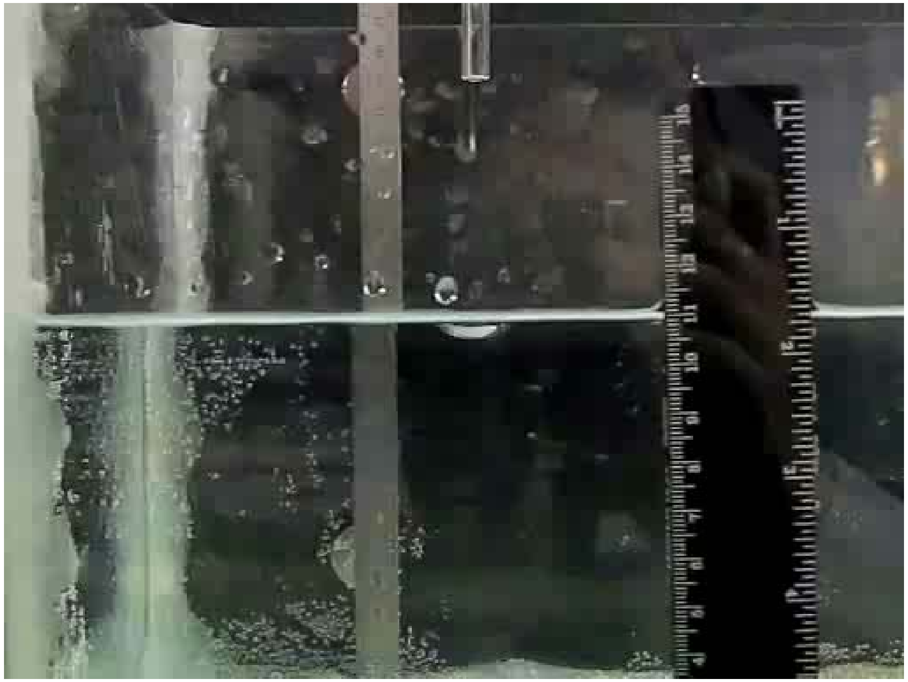
Experimental results – shape



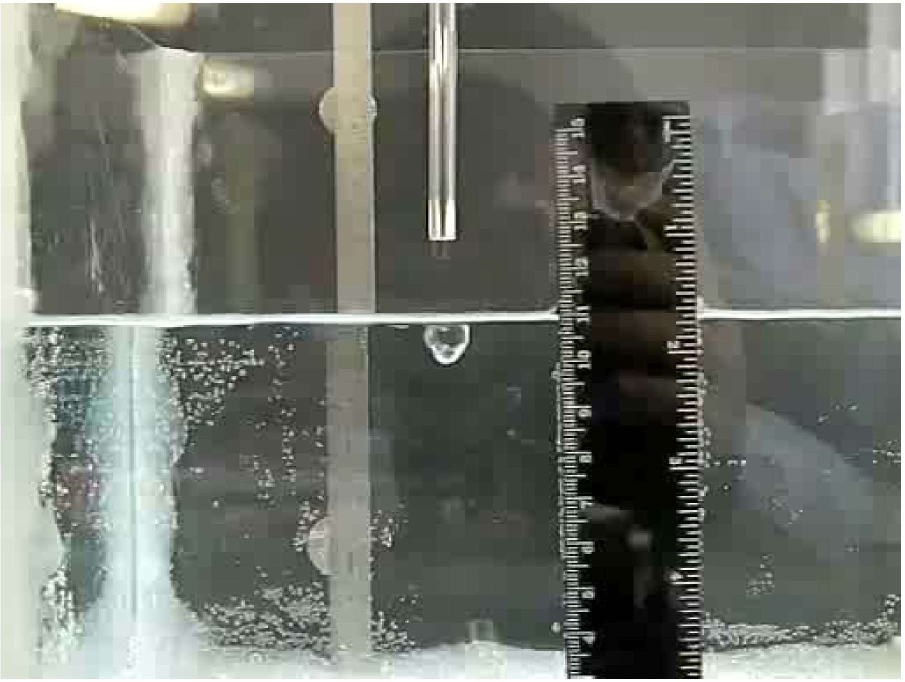
Experimental results – shape



Deep cavities - stability

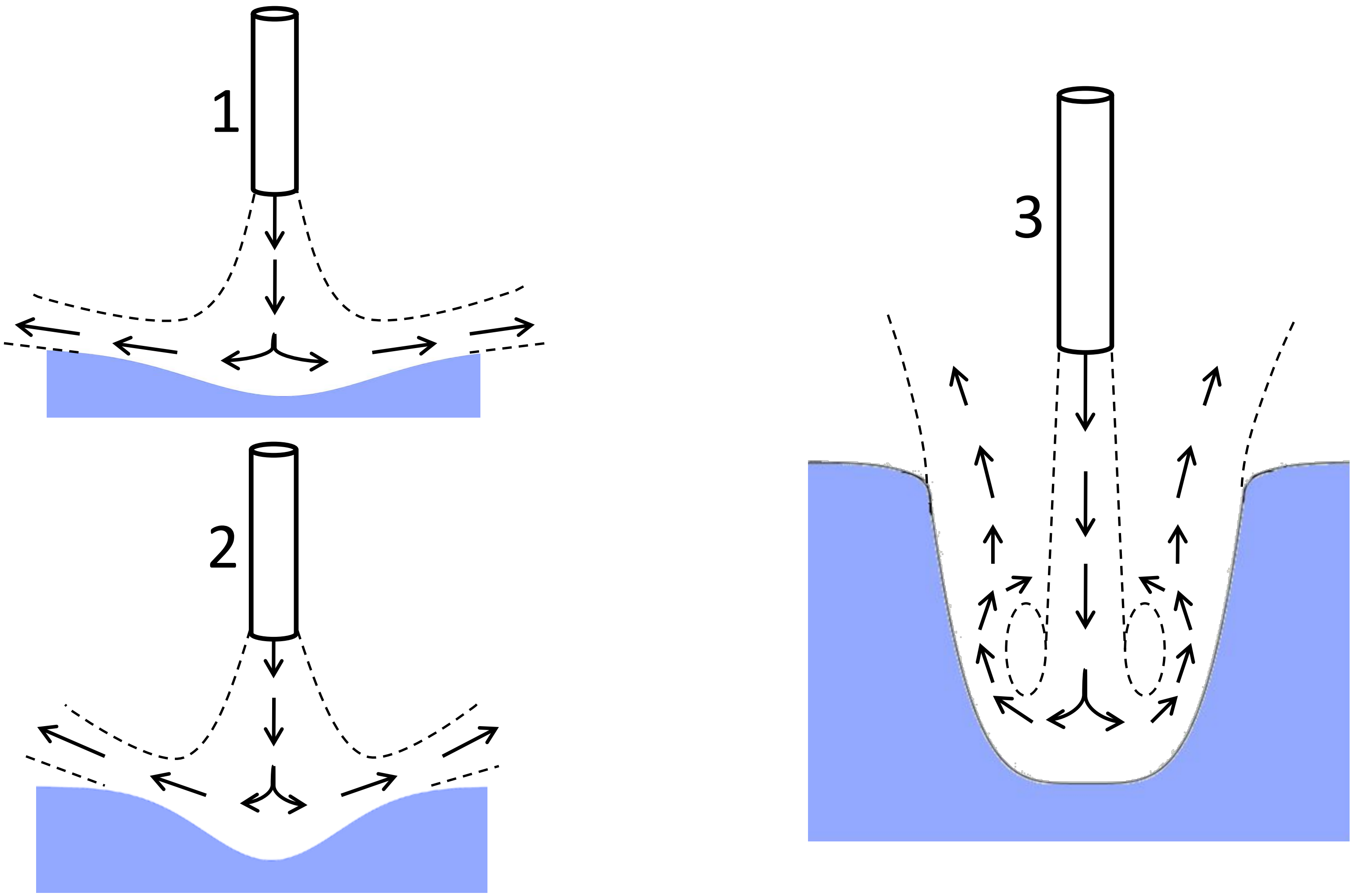


H = 5.93 cm; v_j = 20 m/s; d = 4.6 mm

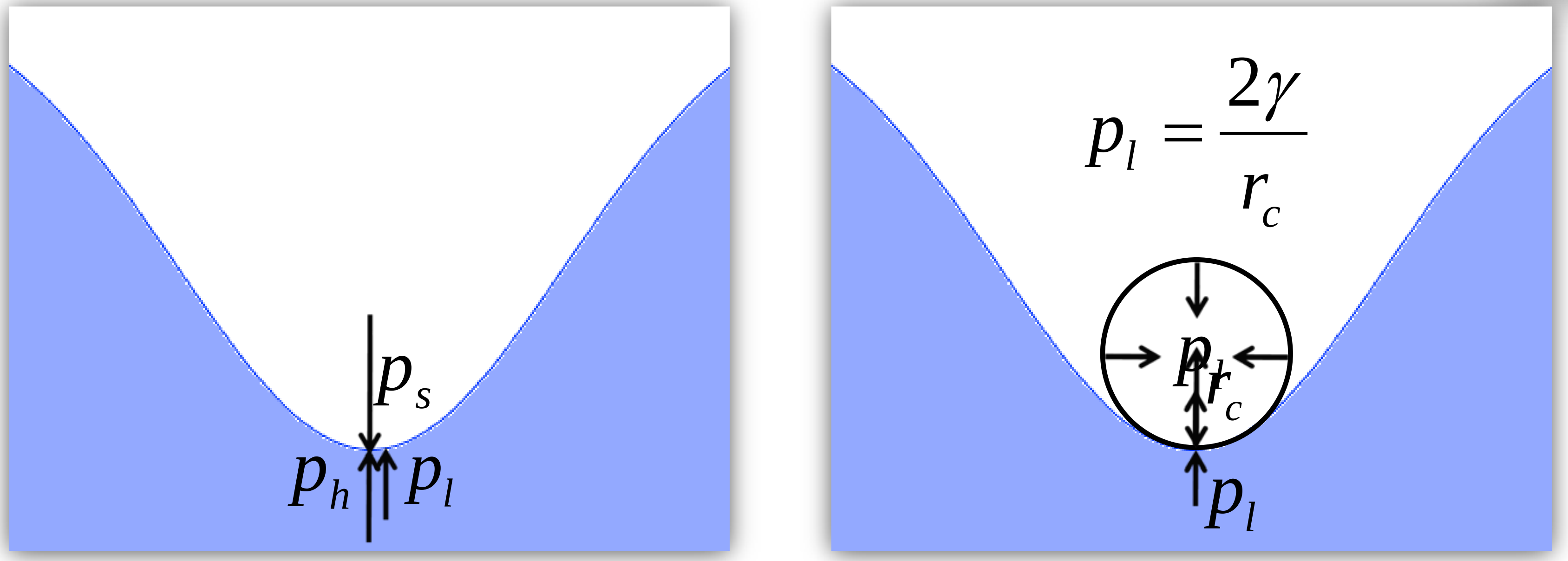


H = 2.23 cm; v_j = 20m/s; d = 4.6 mm

Deep cavities – air flow



Deep cavities – surface tension



$$p_s = p_h + p_l$$

$$\underbrace{\frac{1}{2}\cdot\left(v_j \cdot K \cdot \frac{d}{H+n_0}\right)^2 \cdot \rho_g}_{p_s} = \underbrace{n_0 \cdot g \cdot \rho_W}_{p_h} + \underbrace{\frac{2\gamma}{r_c}}_{p_l}$$

Conclusions

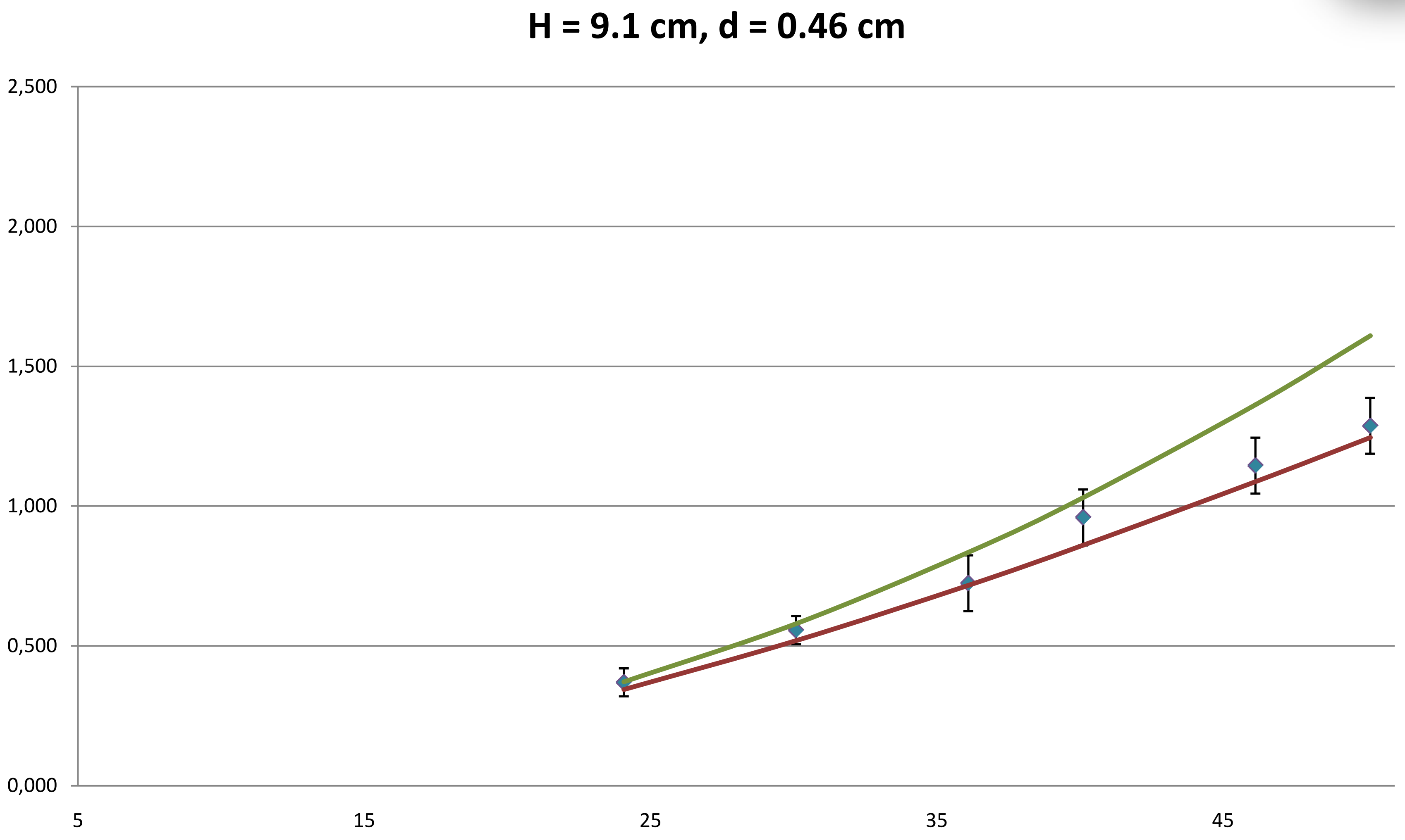
- **Relevant** parameters are:
  - Jet exit velocity (v_j)
  - Diameter of the straw (d)
  - Distance between the straw an the water surface (H)
- Other parameters
  - Air density
  - Water density
  - Surface tension
- Experimental investigation of the relevant parameters
- Model could be extended by:
  - Computing the flow with a CFD system
  - Taking into account surface tension

References

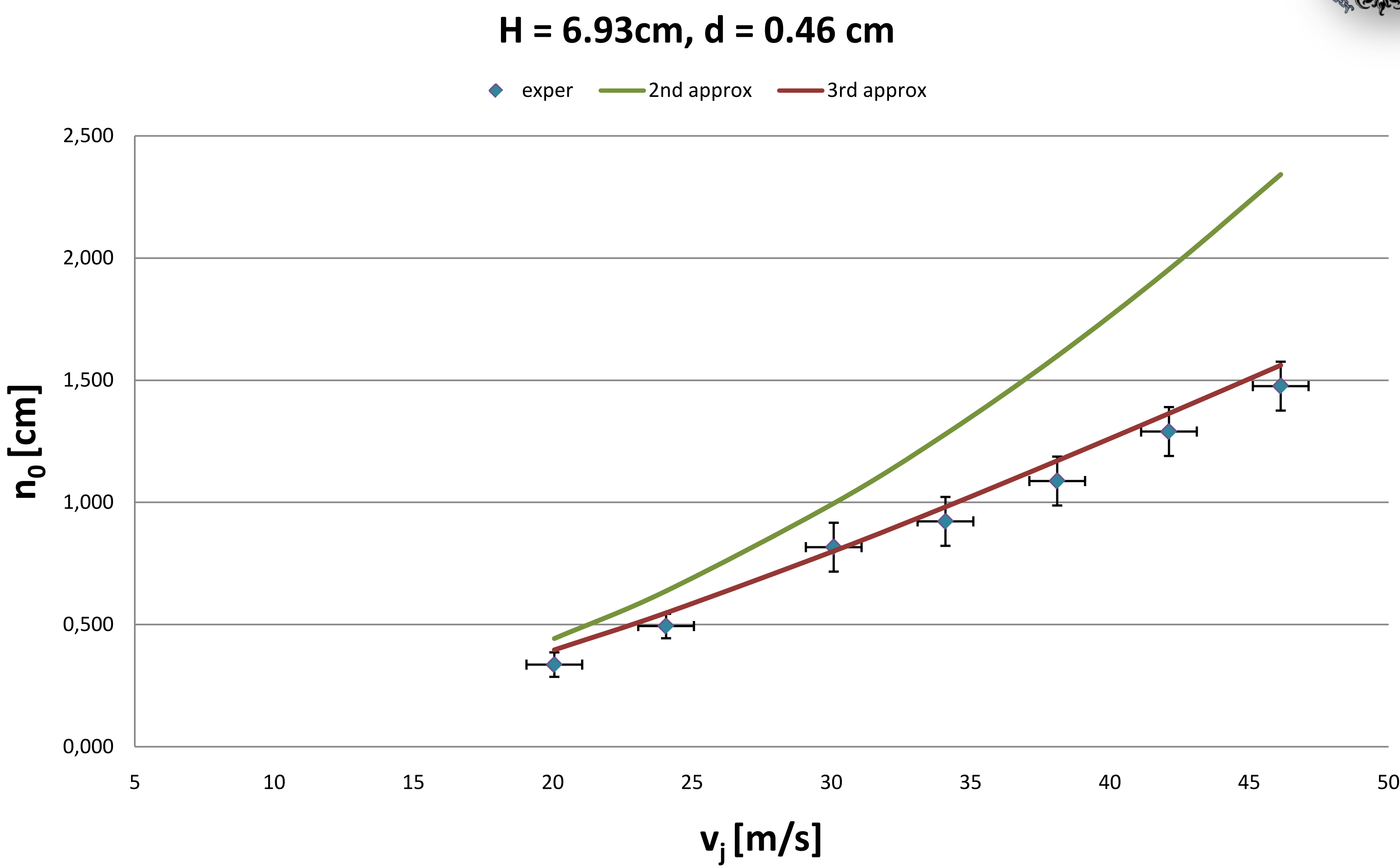
- B. Banks, D.V. Chandrasekhara, J. Fluid Mech.15, p13 (1962)
- R.D. Collins, H.Lubanska, Brit. J. of Applied Physics,Vol.5, p.22(1956)
- F.R. Cheslak, J.A. Nicholls, M. Sichel, J. Fluid Mech.36, p.55(1969)
- Forstall W. & Gaylord E.W., J. Apll. Mech. 22, 161 (1955)
- Albertson et al., Trans. Amer. Soc. Civil Engrs., 155, 639.(1950)
- Poreh M. & Cermak J.E., Proc. Sixth Midwest Conf. on Fluid Mech., Austin p.198 (1959)

Zusatzfolien

Results

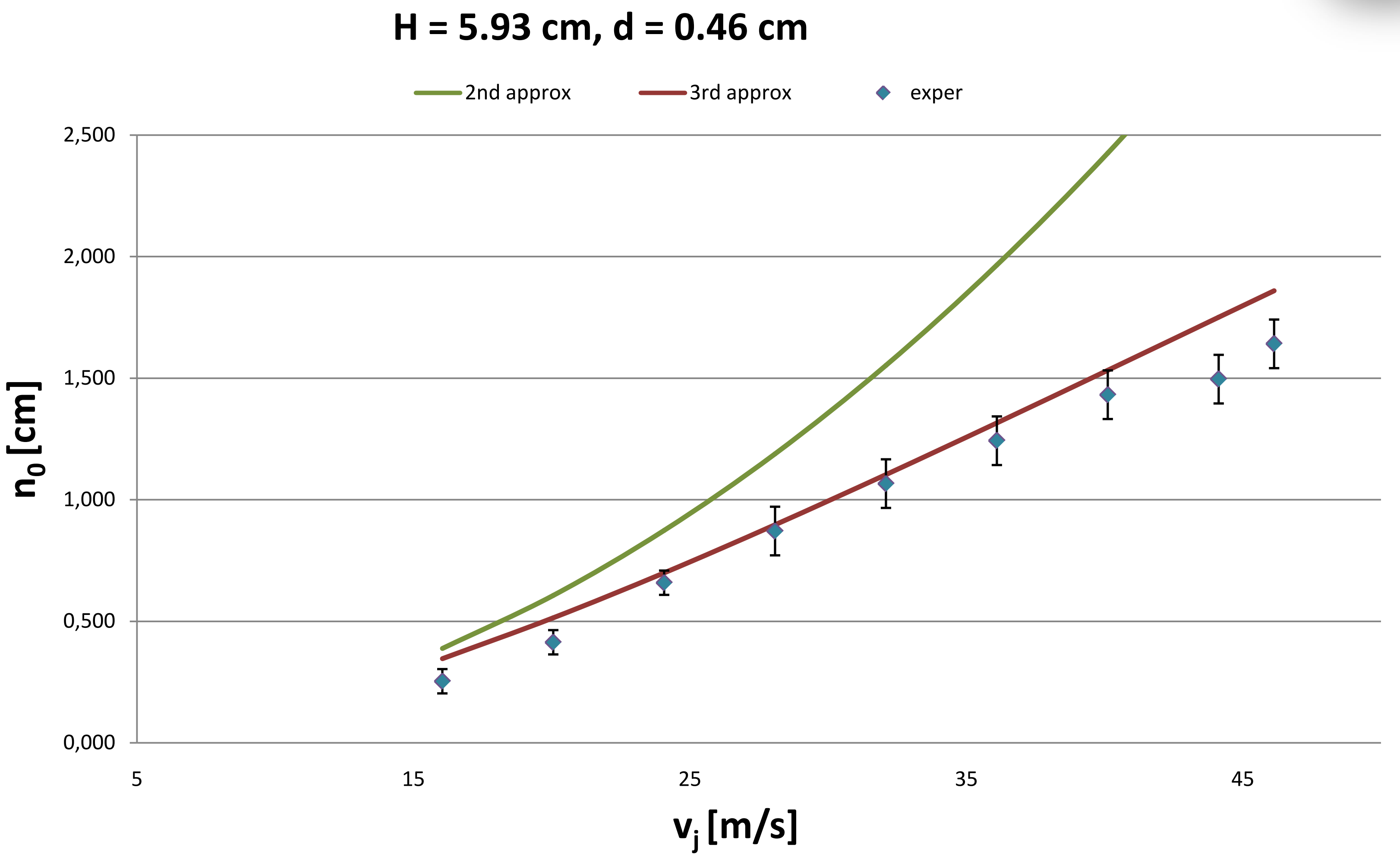


Results



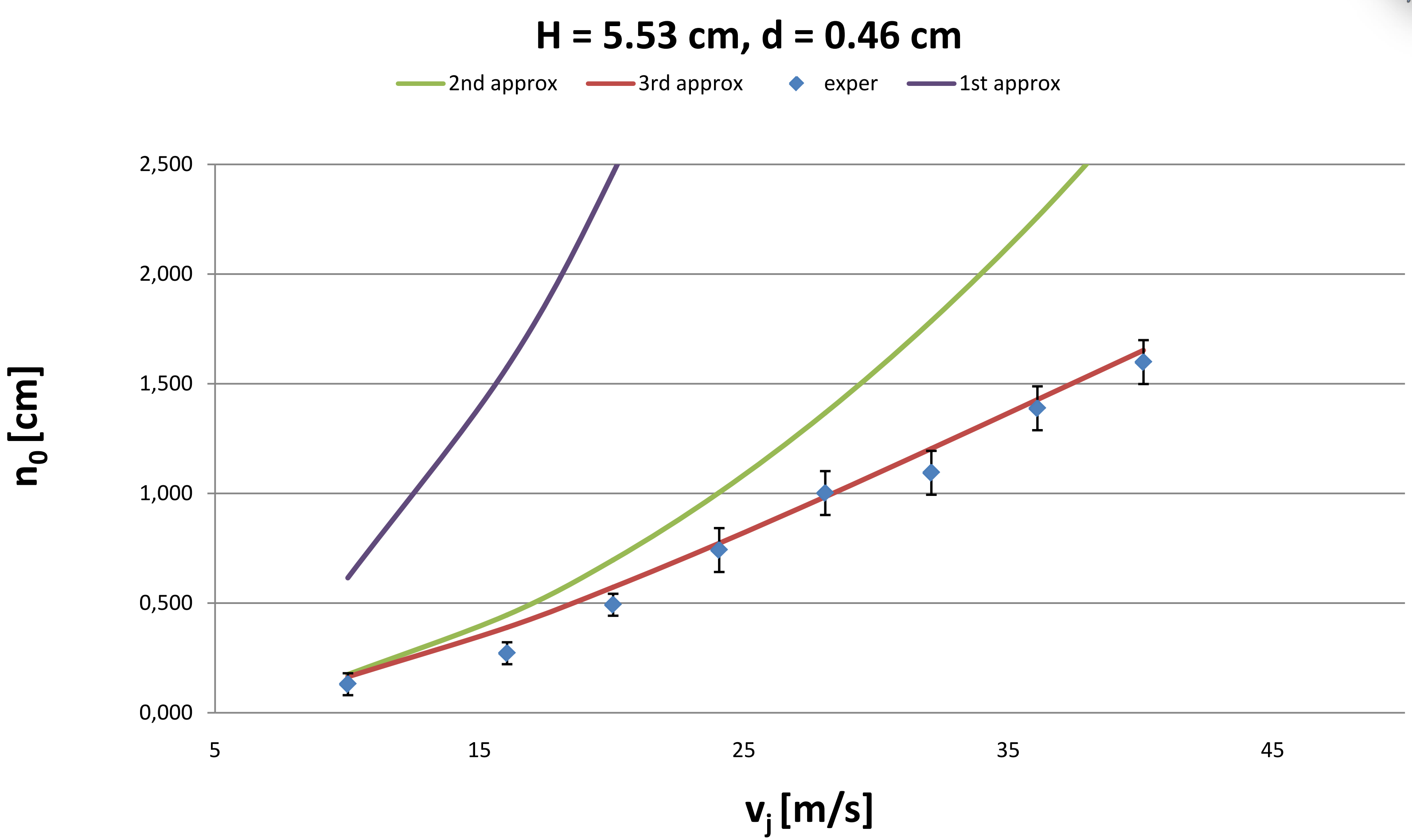
H = 6.93cm, d = 0.46 cm
exper
2nd approx
3rd approx
2,500
2,000
1,500
1,000
0,500
0,000
n0 [cm]
5
10
15
20
25
30
35
40
45
50
vj [m/s]

Results

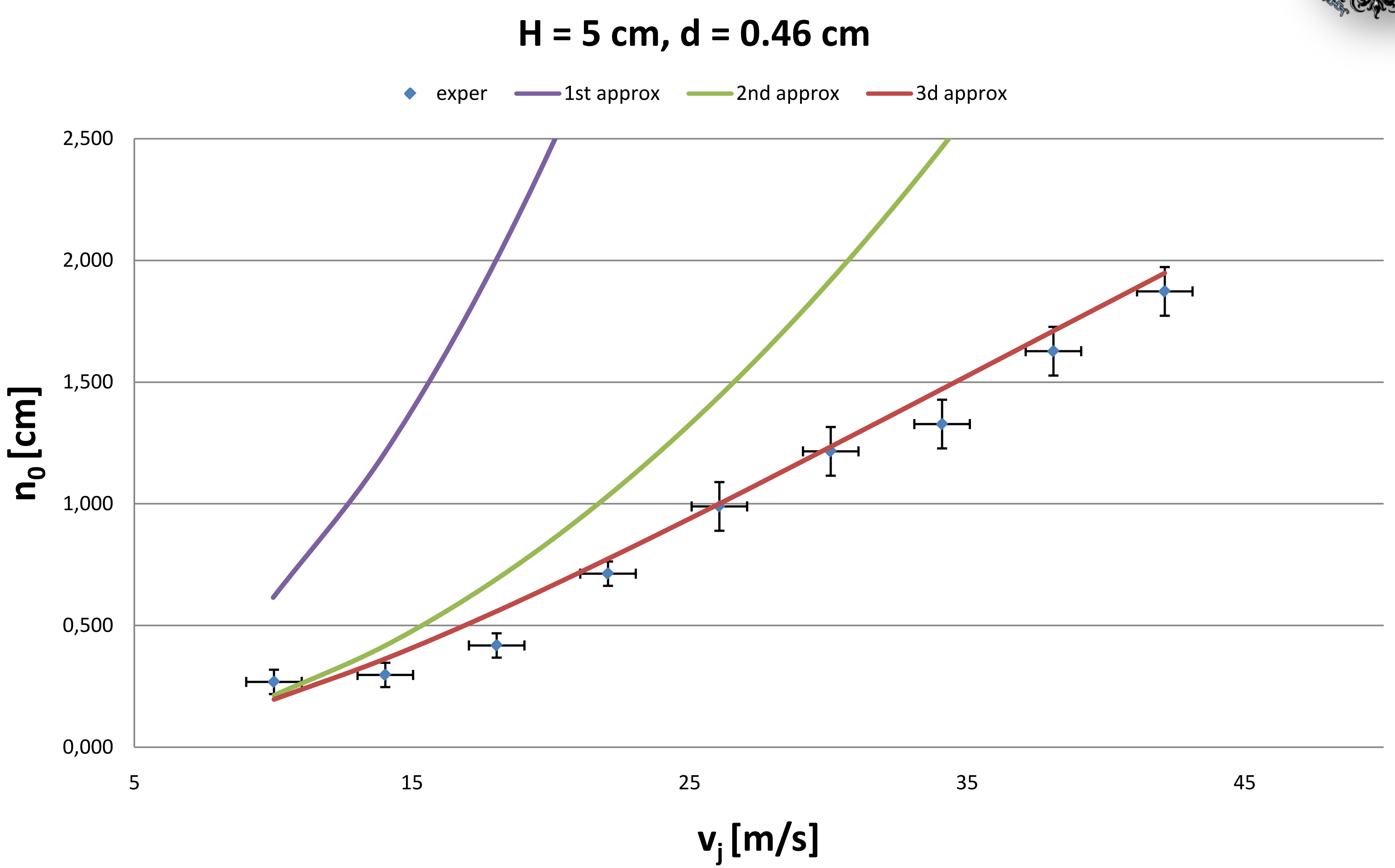


H = 5.93 cm, d = 0.46 cm
2nd approx
3rd approx
exper
2,500
2,000
1,500
1,000
0,500
0,000
5
15
25
35
45
n_0 [cm]
v_j [m/s]

Results

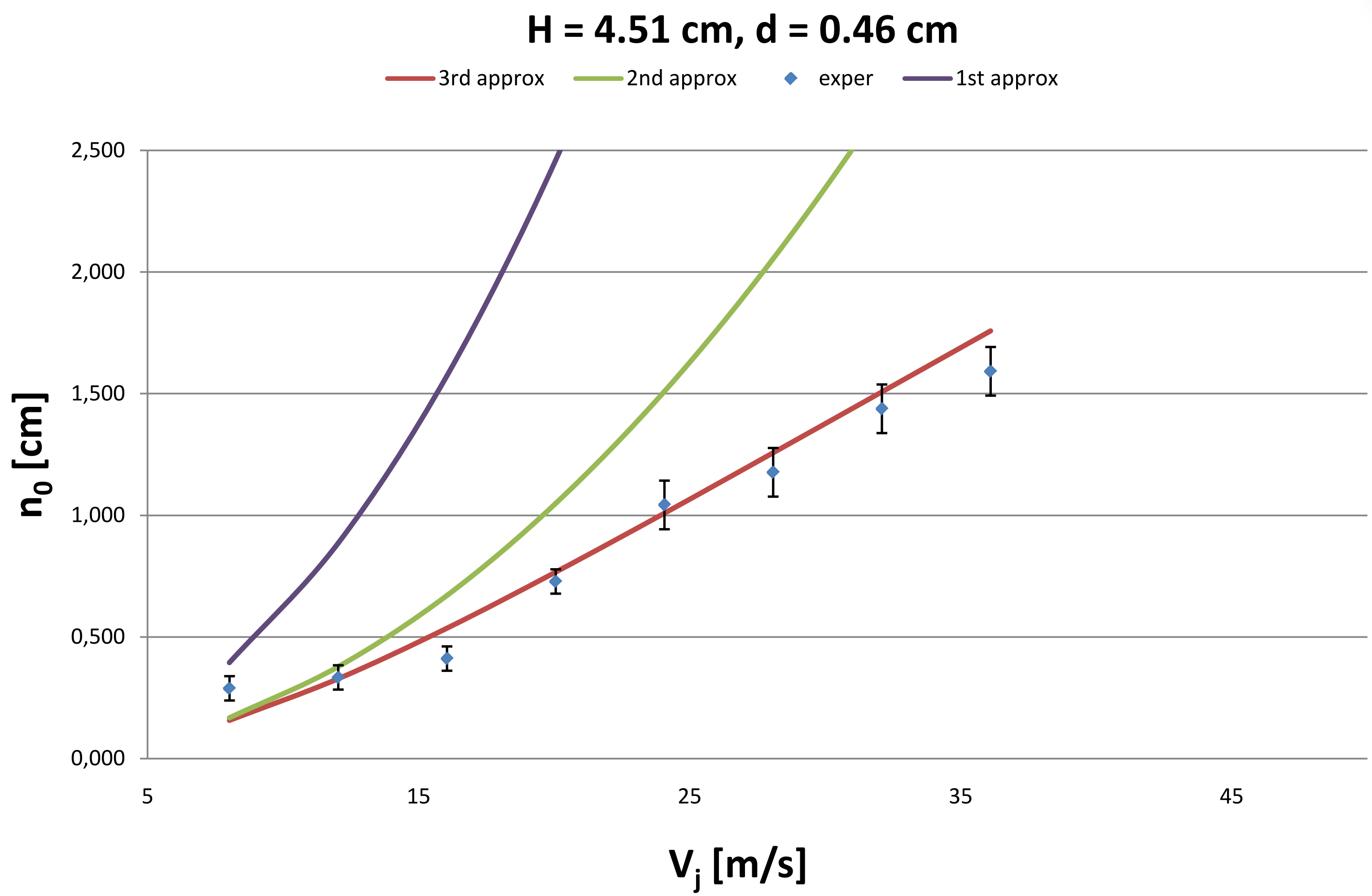

H = 5.53 cm, d = 0.46 cm
2nd approx
3rd approx
exper
1st approx
2,500
2,000
1,500
1,000
0,500
0,000
5
15
25
35
45
n_0 [cm]
v_j [m/s]

Results



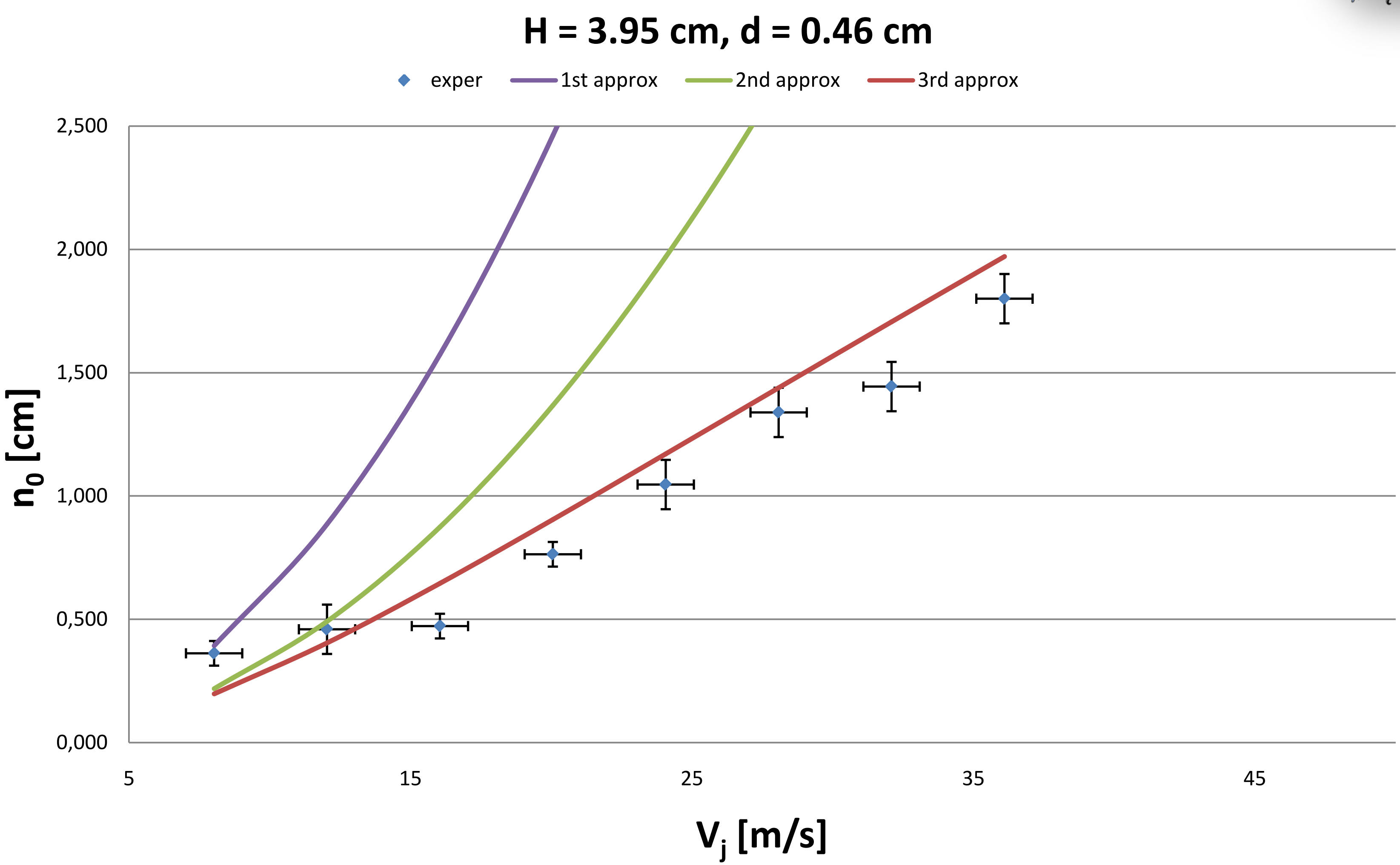
H = 5 cm, d = 0.46 cm
exper
1st approx
2nd approx
3d approx
2,500
2,000
1,500
1,000
0,500
0,000
5
15
25
35
45
n_0 [cm]
v_j [m/s]

Results

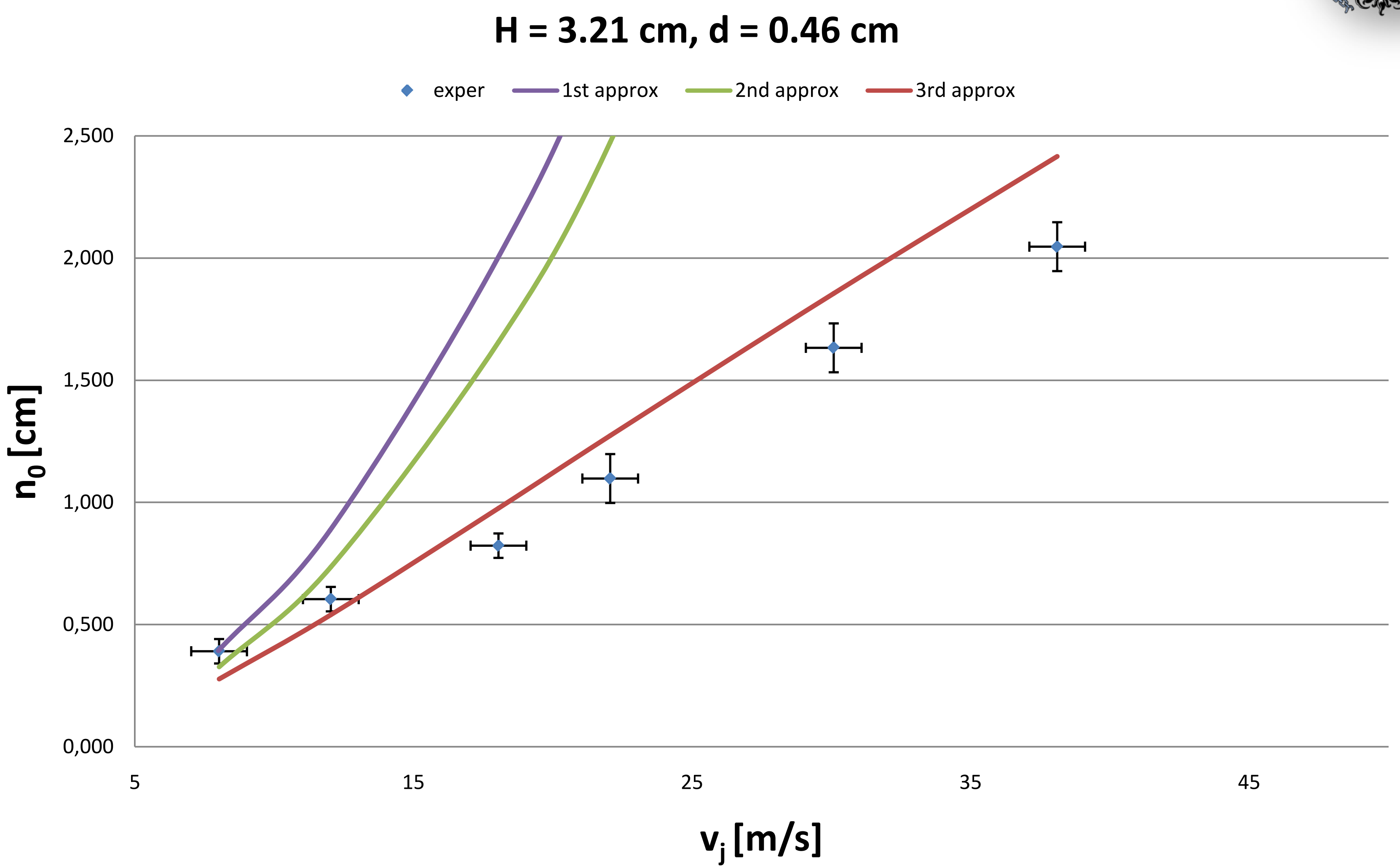


H = 4.51 cm, d = 0.46 cm
3rd approx
2nd approx
exper
1st approx
2,500
2,000
1,500
1,000
0,500
0,000
5
15
25
35
45
n_0 [cm]
V_j [m/s]

Results

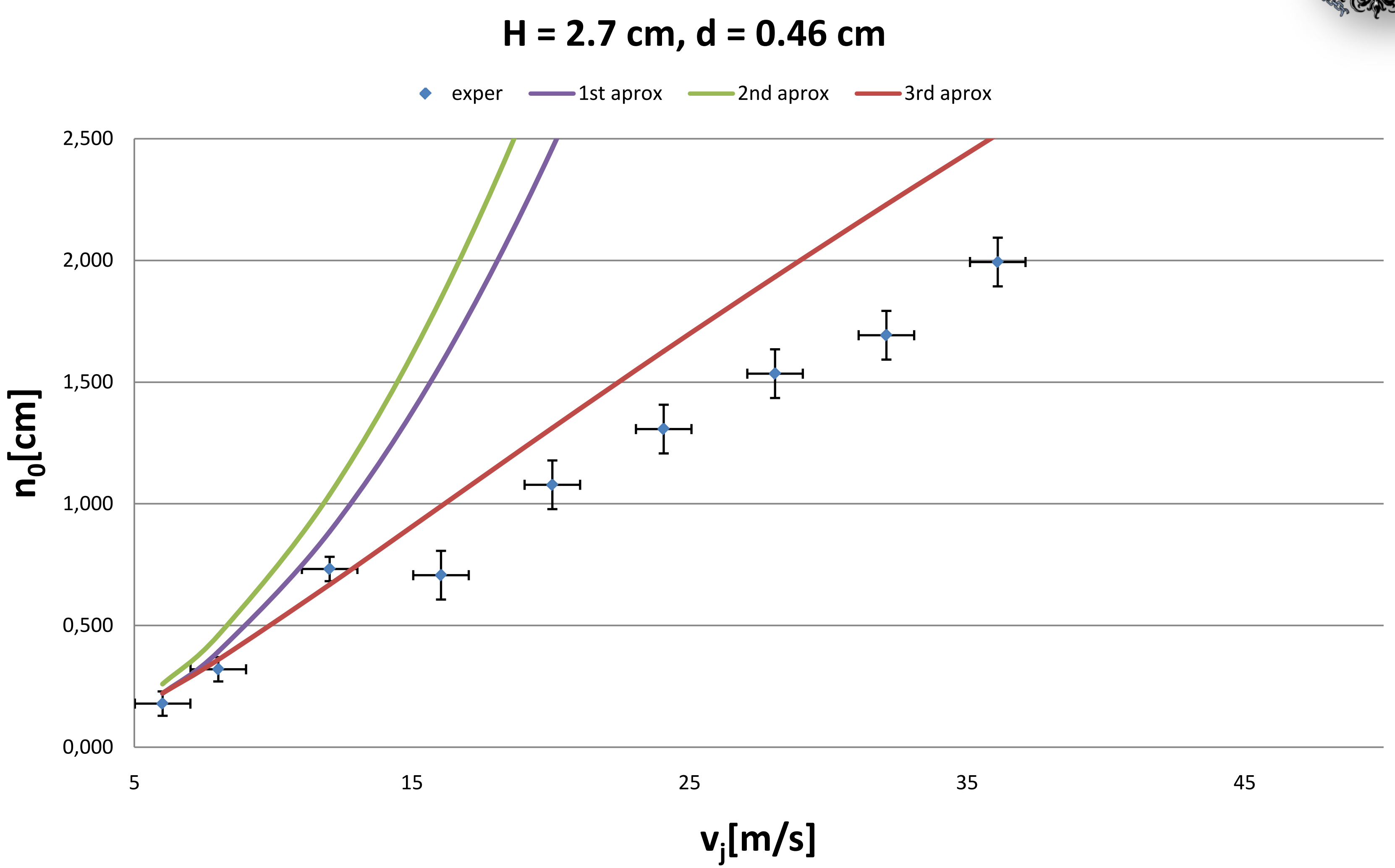

H = 3.95 cm, d = 0.46 cm
exper
1st approx
2nd approx
3rd approx
2,500
2,000
1,500
1,000
0,500
0,000
n0 [cm]
5
15
25
35
45
Vj [m/s]

Results



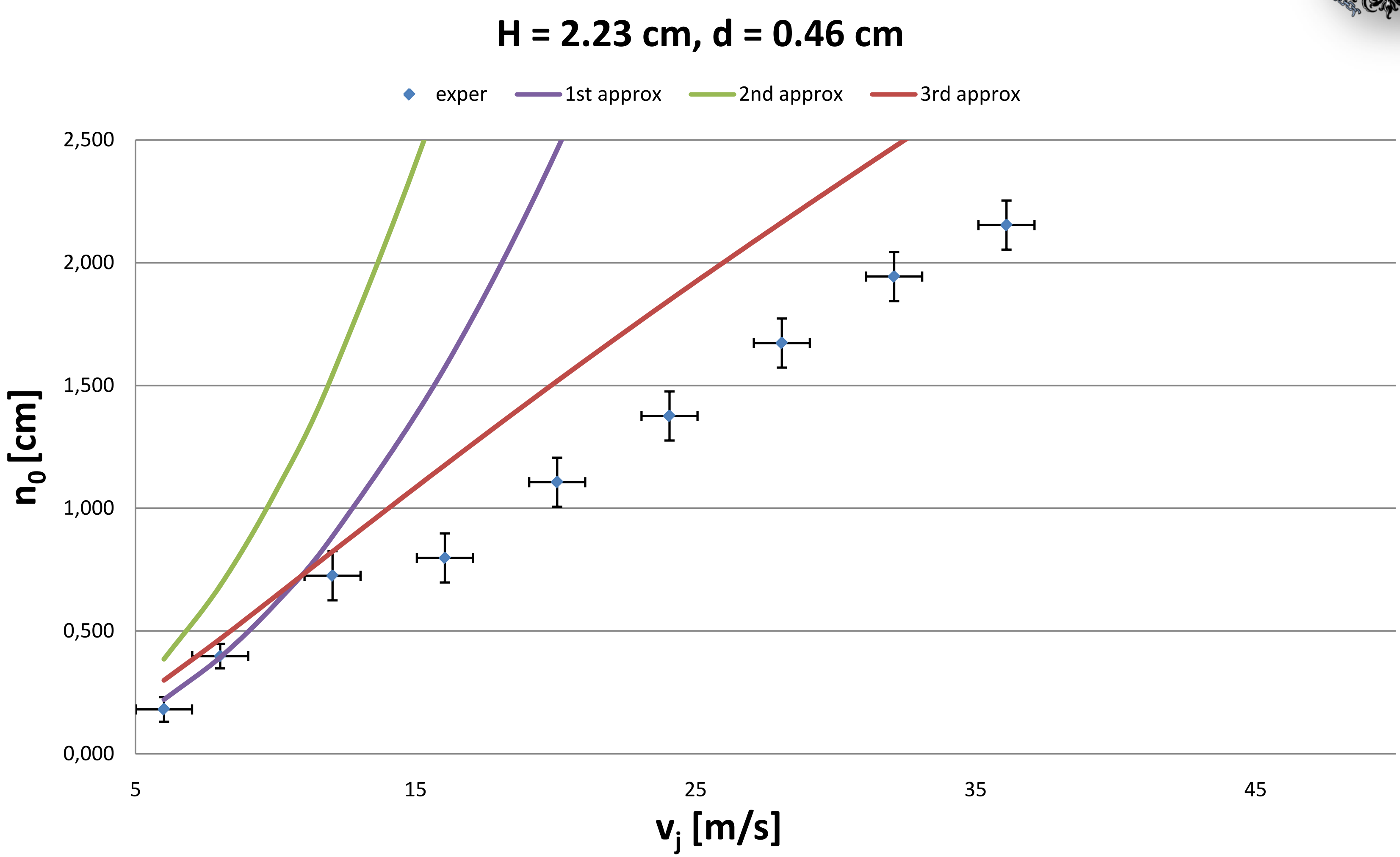
H = 3.21 cm, d = 0.46 cm
exper
1st approx
2nd approx
3rd approx
2,500
2,000
1,500
1,000
0,500
0,000
n0 [cm]
5
15
25
35
45
vj [m/s]

Results



H = 2.7 cm, d = 0.46 cm
exper
1st aprox
2nd aprox
3rd aprox
2,500
2,000
1,500
1,000
0,500
0,000
n_0[cm]
5
15
25
35
45
v_j[m/s]

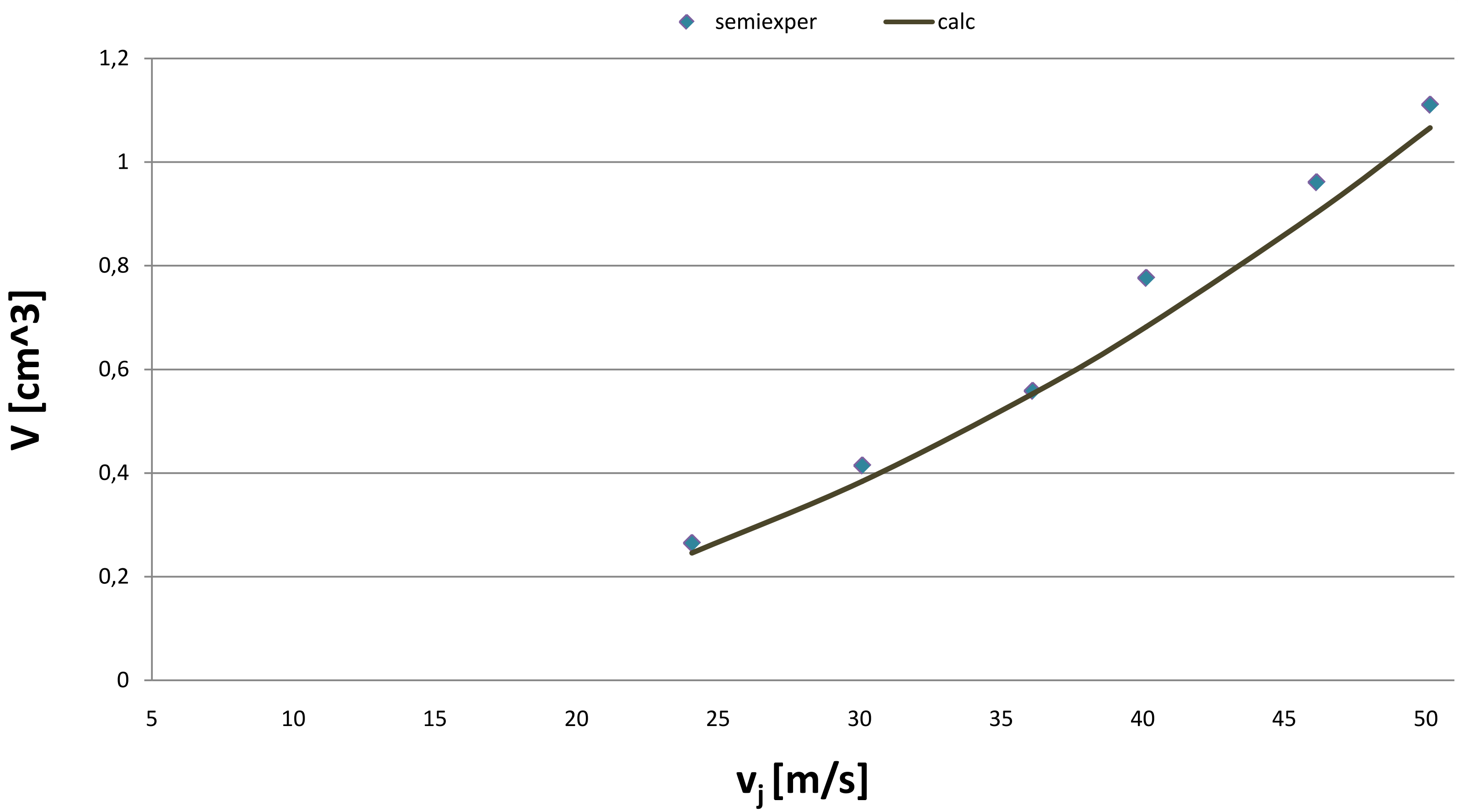
Results



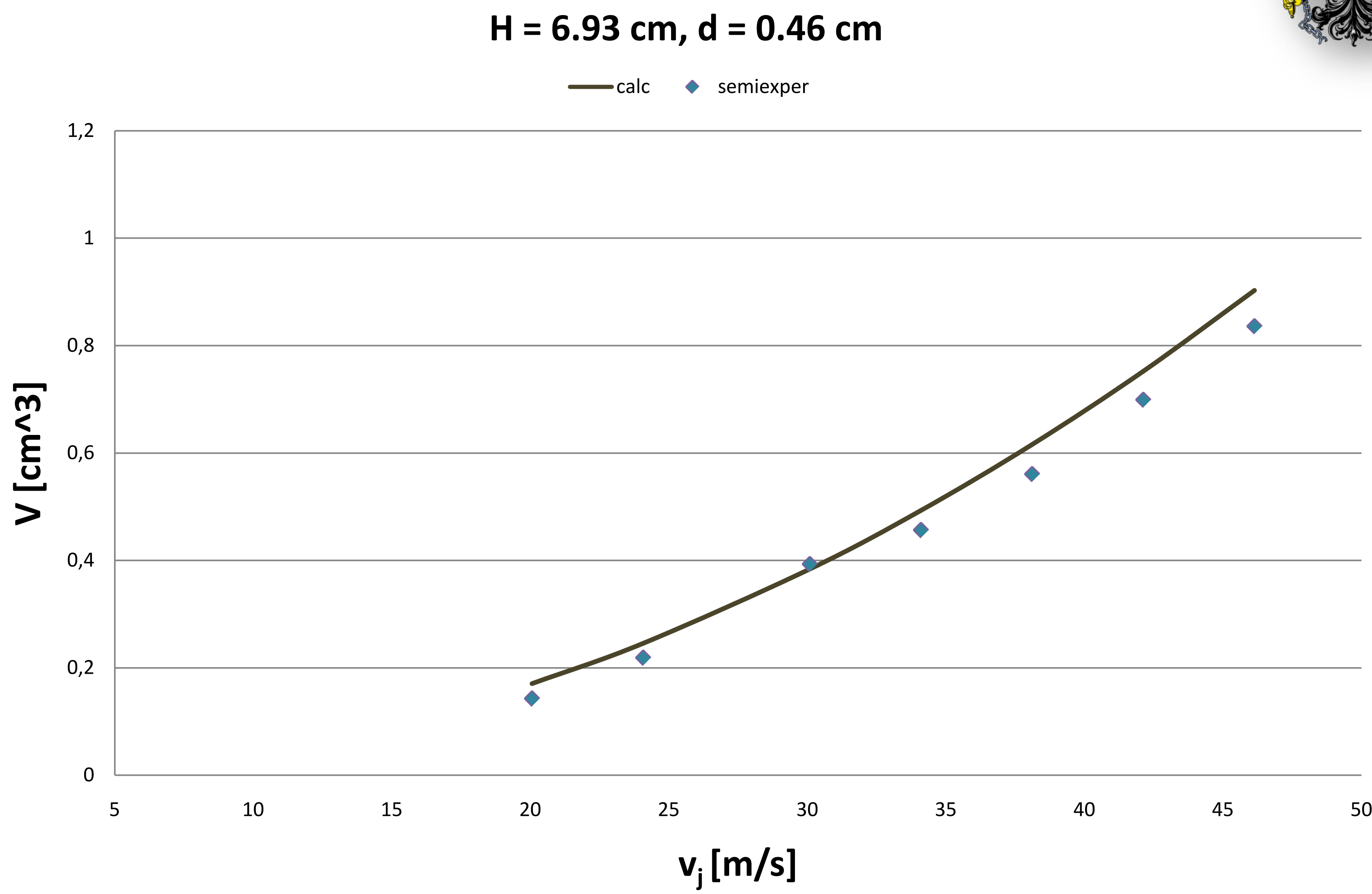
H = 2.23 cm, d = 0.46 cm
exper
1st approx
2nd approx
3rd approx
2,500
2,000
1,500
1,000
0,500
0,000
n0 [cm]
5
15
25
35
45
vj [m/s]

Results

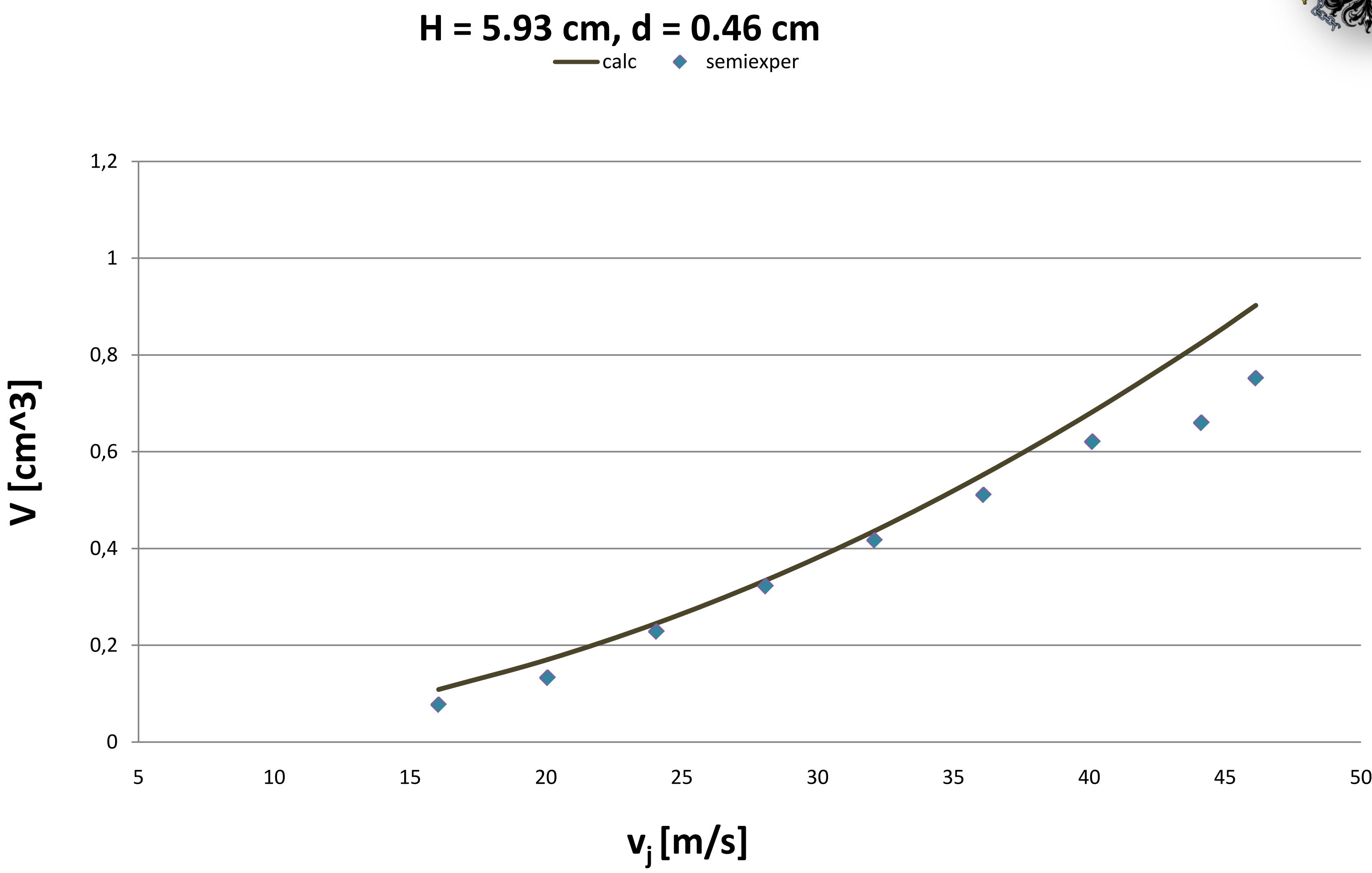
H = 9.1 cm, d = 0.46 cm



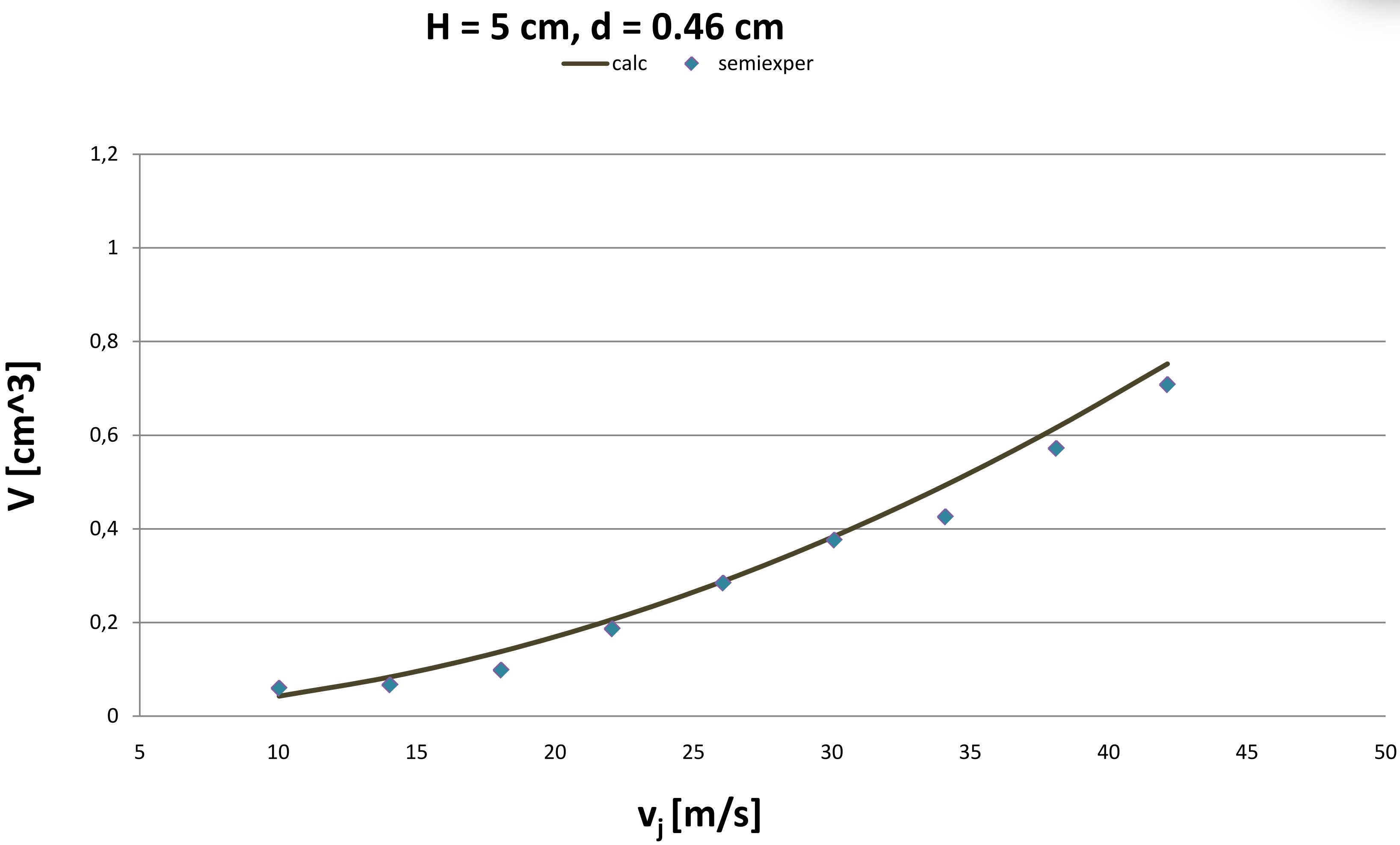
Results


H = 6.93 cm, d = 0.46 cm
calc
semiexper
1,2
1
0,8
0,6
0,4
0,2
0
V [cm^3]
5
10
15
20
25
30
35
40
45
50
v_j [m/s]

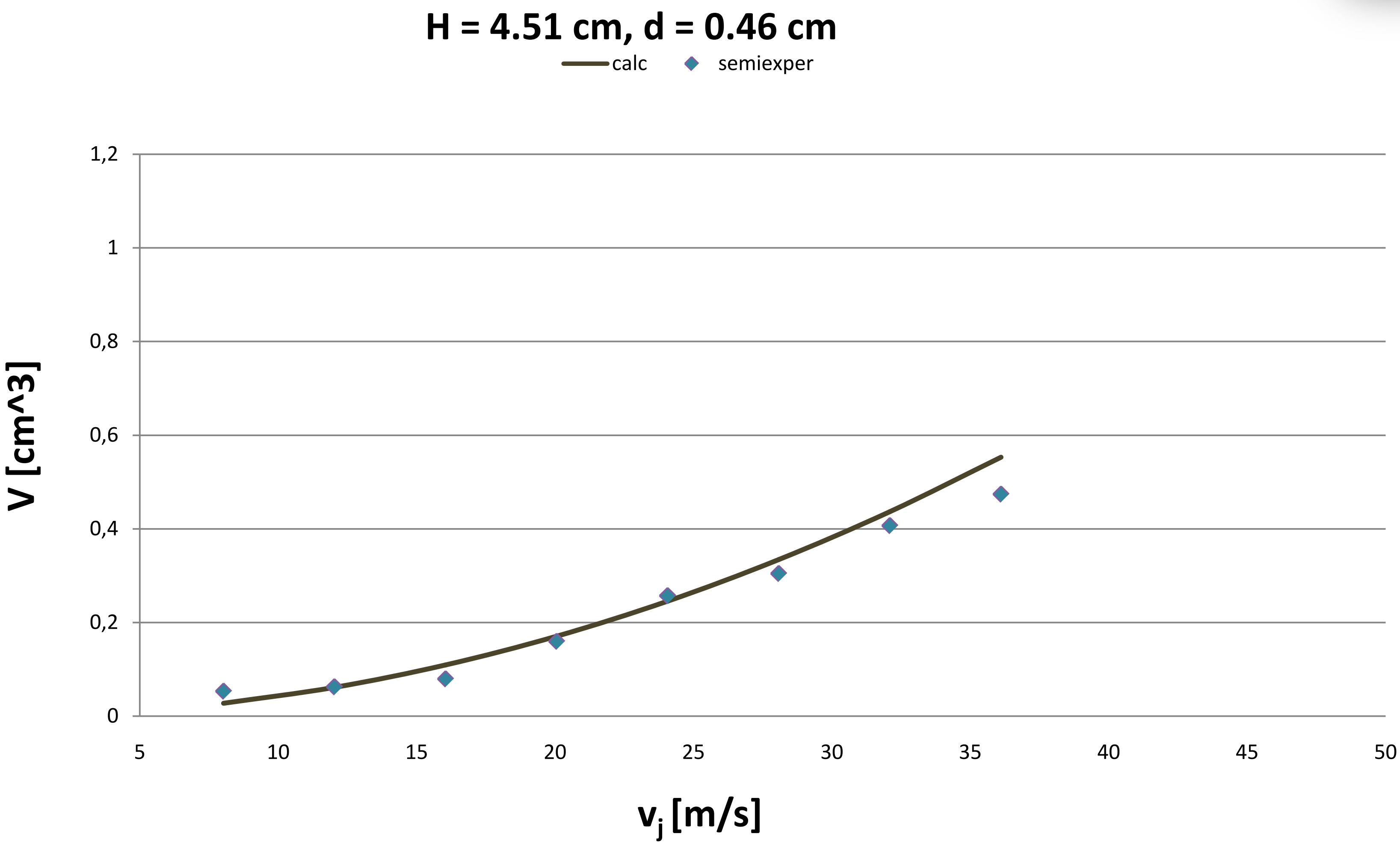
Results


H = 5.93 cm, d = 0.46 cm
calc
semiexper
1,2
1
0,8
0,6
0,4
0,2
0
V [cm^3]
5
10
15
20
25
30
35
40
45
50
vj [m/s]

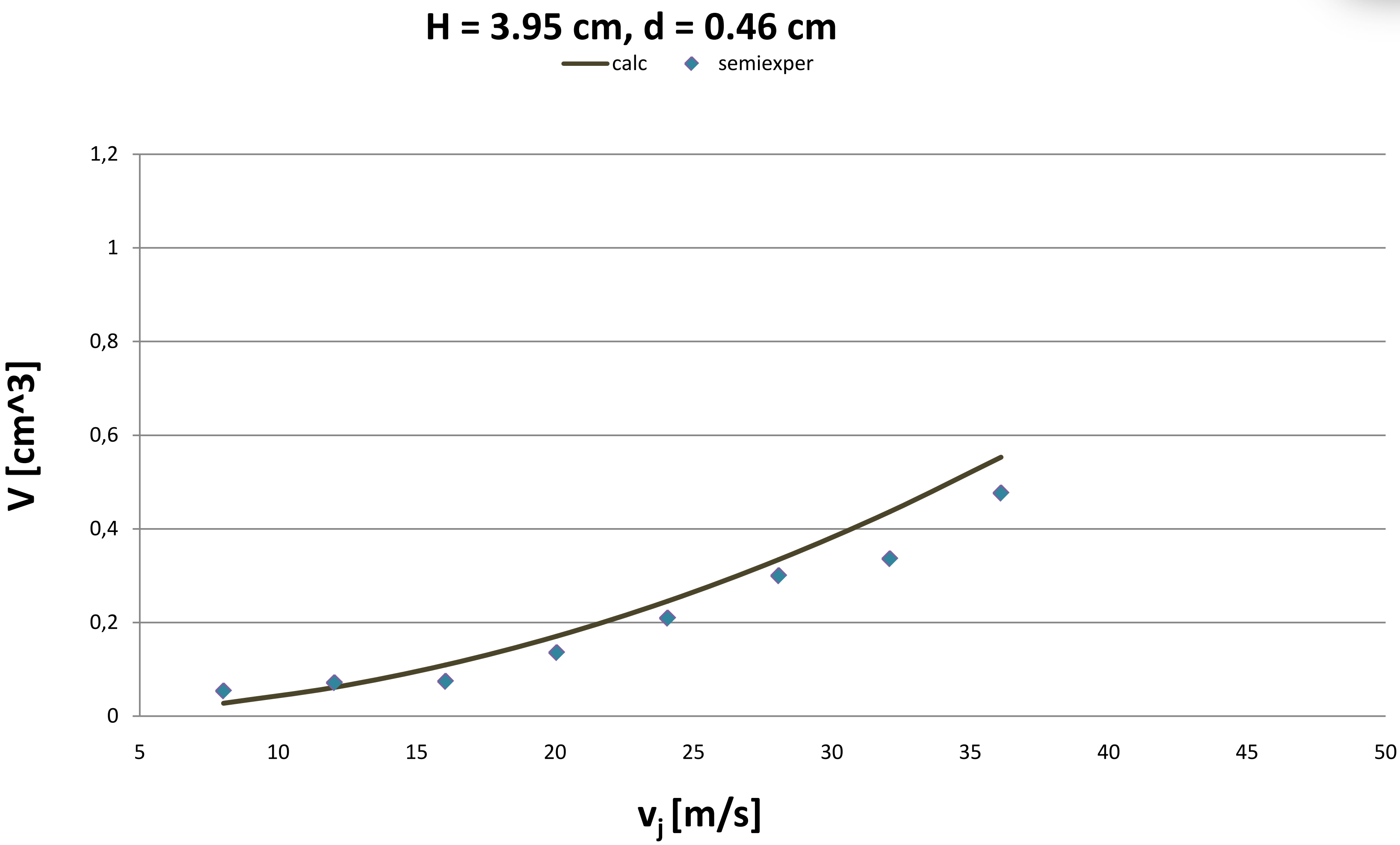
Results


H = 5 cm, d = 0.46 cm
calc
semiexper
1,2
1
0,8
0,6
0,4
0,2
0
5
10
15
20
25
30
35
40
45
50
V [cm^3]
v_j [m/s]

Results

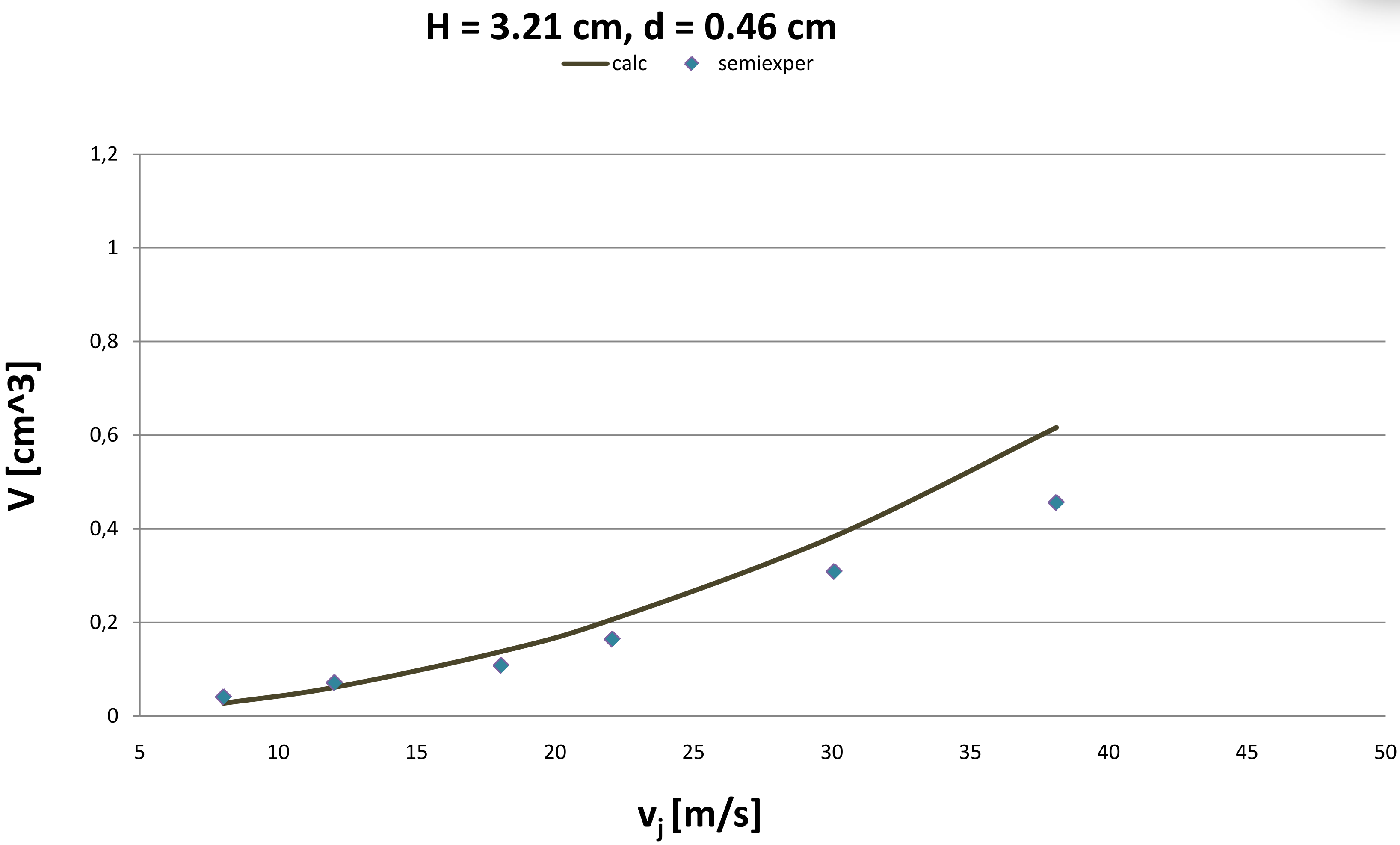

H = 4.51 cm, d = 0.46 cm
calc
semiexper
1,2
1
0,8
0,6
0,4
0,2
0
V [cm^3]
5
10
15
20
25
30
35
40
45
50
v j [m/s]

Results

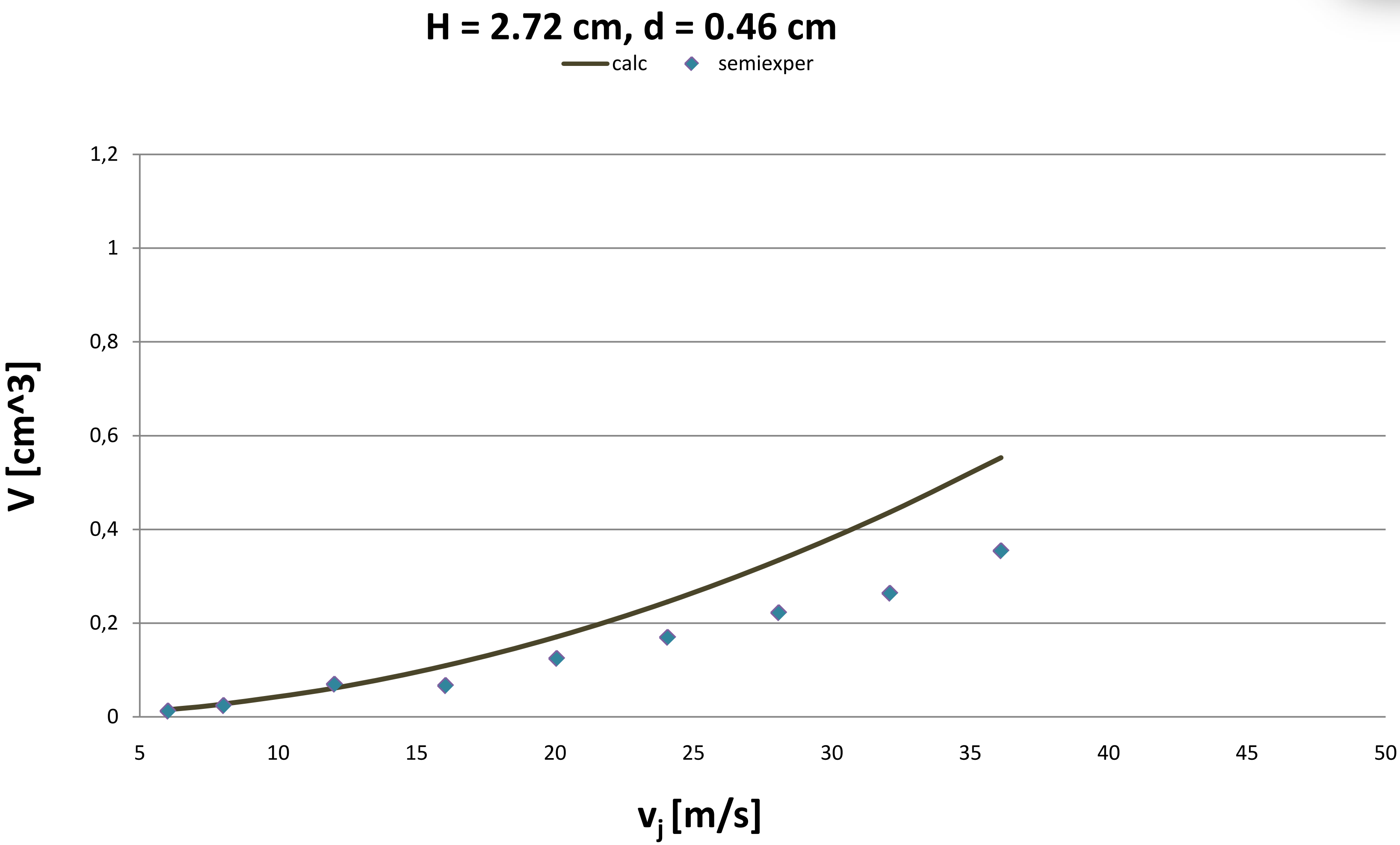


H = 3.95 cm, d = 0.46 cm
calc
semiexper
1,2
1
0,8
0,6
0,4
0,2
0
V [cm^3]
5
10
15
20
25
30
35
40
45
50
v_j [m/s]

Results

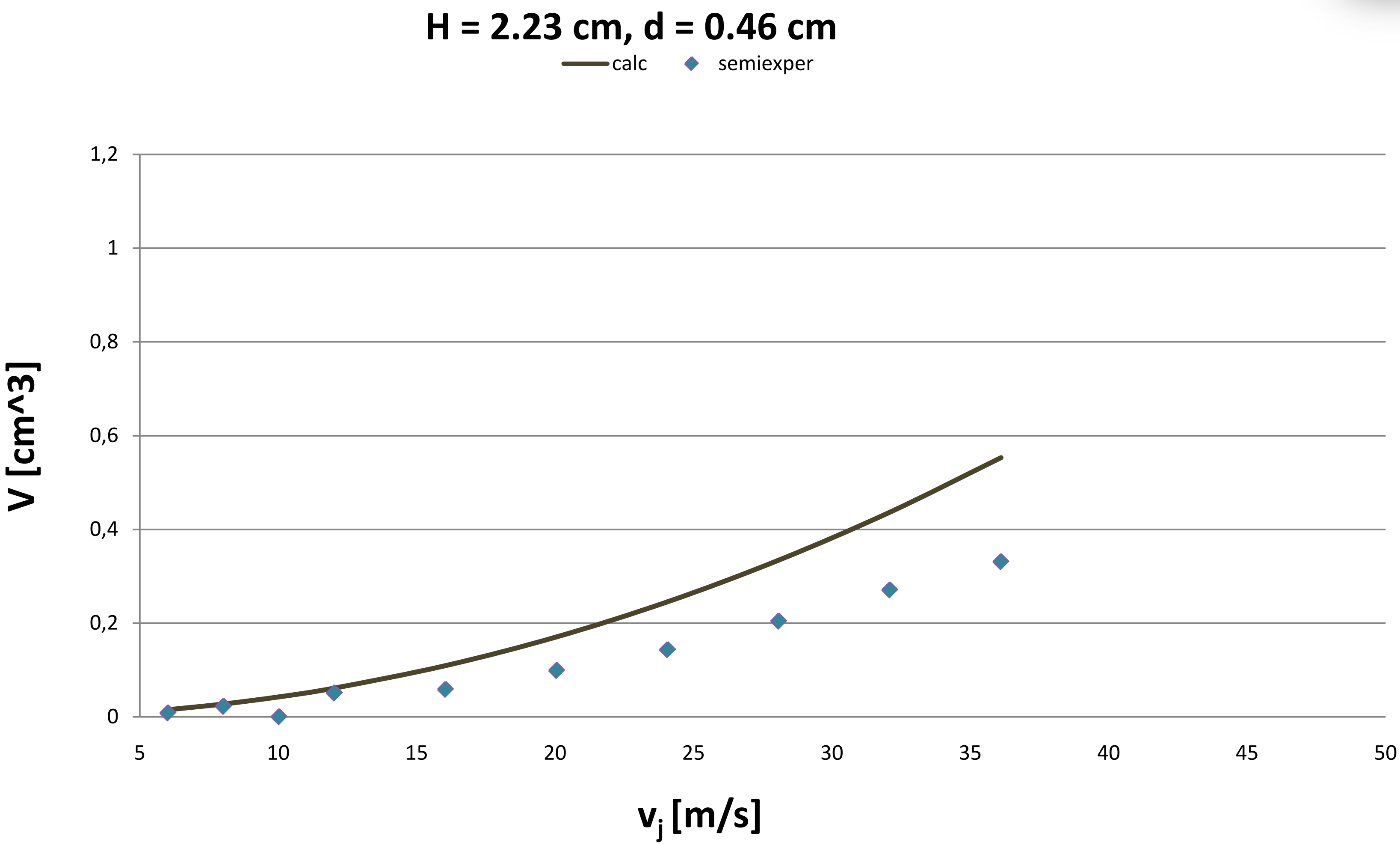

H = 3.21 cm, d = 0.46 cm
calc
semiexper
1,2
1
0,8
0,6
0,4
0,2
0
V [cm^3]
5
10
15
20
25
30
35
40
45
50
v_j [m/s]

Results



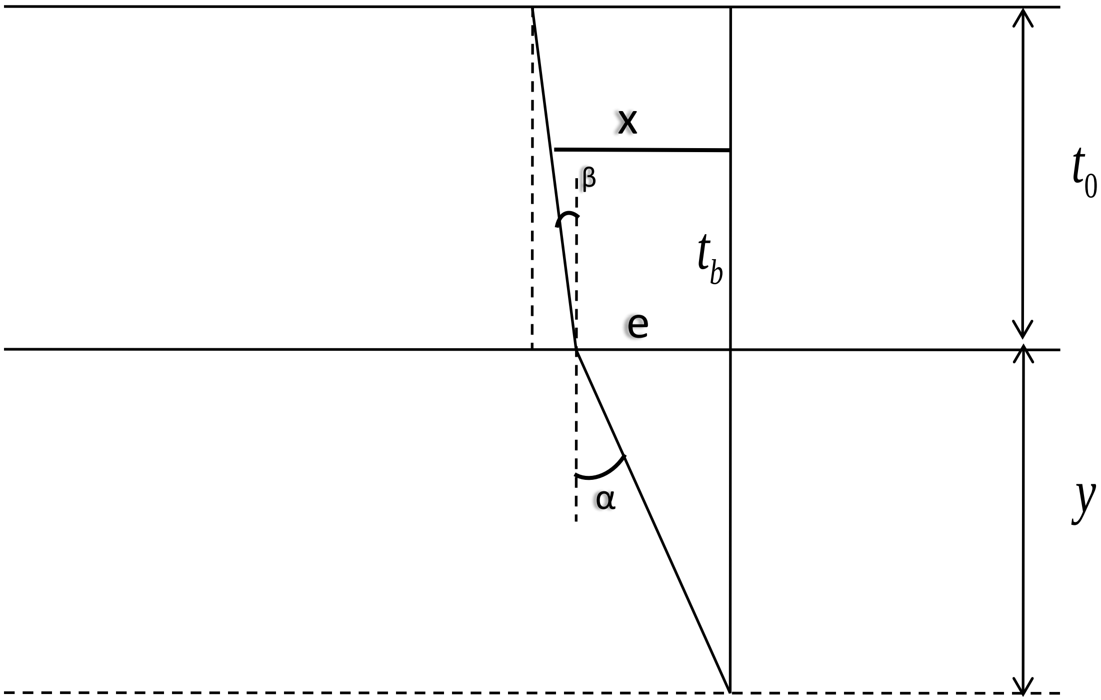
H = 2.72 cm, d = 0.46 cm
calc
semiexper
1,2
1
0,8
0,6
0,4
0,2
0
V [cm^3]
5
10
15
20
25
30
35
40
45
50
v_j [m/s]

Results



H = 2.23 cm, d = 0.46 cm
calc
semiexper
1,2
1
0,8
0,6
0,4
0,2
0
V [cm^3]
5
10
15
20
25
30
35
40
45
50
v_j [m/s]

Distortion due to refraction



$$x = e + t_b \cdot \tan\beta$$

$$y \tan\alpha = e$$

$$\sin\alpha = n \cdot \sin\beta$$

$$\tan\alpha = \frac{e}{y} \qquad n = 1.33$$

$$x = e + t_b \cdot \tan\left\{ \arcsin\left[\frac{\sin\left(\arctan\left(\frac{e}{y} \right) \right)}{n} \right] \right\}$$

3rd approximation - formula

$$n_0 = \frac{1}{6}\,\frac{\left(\left(8\,H^3\,\rho_W g + 54\,\rho_G V_J^2 K_2^2 d_0^2 + 6\sqrt{3}\,V_J K_2 d_0\sqrt{\rho_G\left(8\,H^3\,\rho_W g + 27\,\rho_G V_J^2 K_2^2 d_0^2\right)}\right)\rho_W^2 g^2\right)^{1/3}}{\rho_W g} + \frac{2}{3}\,\frac{H^2\,\rho_W g}{\left(\left(8\,H^3\,\rho_W g + 54\,\rho_G V_J^2 K_2^2 d_0^2 + 6\sqrt{3}\,V_J K_2 d_0\sqrt{\rho_G\left(8\,H^3\,\rho_W g + 27\,\rho_G V_J^2 K_2^2 d_0^2\right)}\right)\rho_W^2 g^2\right)^{1/3}} - \frac{2}{3}\,H$$